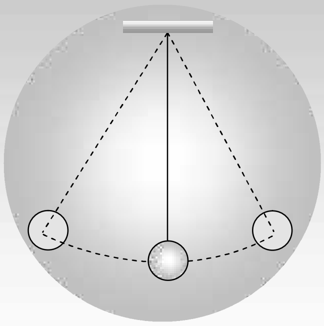


Simple Harmonic Motion

15.1 Periodic Motion

A motion, which repeat itself over and over again after a regular interval of time is called a periodic motion and the fixed interval of time after which the motion is repeated is called period of the motion.

Examples :

(i) Revolution of earth around the sun (period one year)

(ii) Rotation of earth about its polar axis (period one day)

(iii) Motion of hour's hand of a clock (period 12-hour)

(iv) Motion of minute's hand of a clock (period 1-hour)

(v) Motion of second's hand of a clock (period 1-minute)

(vi) Motion of moon around the earth (period 27.3 days)

15.2 Oscillatory or Vibratory Motion

Oscillatory or vibratory motion is that motion in which a body moves to and fro or back and forth repeatedly about a fixed point in a definite interval of time. In such a motion, the body is confined with in well-defined limits on either side of mean position.

Oscillatory motion is also called as harmonic motion.

Example :

(i) The motion of the pendulum of a wall clock.

(ii) The motion of a load attached to a spring, when it is pulled and then released.

(iii) The motion of liquid contained in U- tube when it is compressed once in one limb and left to itself.

(iv) A loaded piece of wood floating over the surface of a liquid when pressed down and then released executes oscillatory motion.

15.3 Harmonic and Non-harmonic Oscillation

Harmonic oscillation is that oscillation which can be expressed in terms of single harmonic function (*i.e.* sine or cosine function). *Example* : $y = a \sin \mu\, t$ or $y = a \cos \mu\, t$

Non-harmonic oscillation is that oscillation which can not be expressed in terms of single harmonic function. It is a combination of two or more than two harmonic oscillations. *Example* : $y = a \sin \mu\, t + b \sin 2\mu\, t$

15.4 Some Important Definitions

(1) **Time period :** It is the least interval of time after which the periodic motion of a body repeats itself.

S.I. units of time period is second.

(2) **Frequency :** It is defined as the number of periodic motions executed by body per second. S.I unit of frequency is hertz (*Hz*).

(3) **Angular Frequency :** Angular frequency of a body executing periodic motion is equal to product of frequency of the body with factor 2®. Angular frequency μ = 2 ® *n*

S.I. units of μ is *Hz* [S.I.] μ also represents angular velocity. In that case unit will be *rad/sec.*

(4) **Displacement :** In general, the name displacement is given to a physical quantity which undergoes a change with time in a periodic motion.

Examples :

(i) In an oscillation of a loaded spring, displacement variable is its deviation from the mean position.

(ii) During the propagation of sound wave in air, the displacement variable is the local change in pressure

(iii) During the propagation of electromagnetic waves, the displacement variables are electric and magnetic fields, which vary periodically.

(5) **Phase :** phase of a vibrating particle at any instant is a physical quantity, which completely express the position and direction of motion, of the particle at that instant with respect to its mean position.

In oscillatory motion the phase of a vibrating particle is the argument of *sine* or *cosine* function involved to represent the generalised equation of motion of the vibrating particle.

$y = a\sin\theta = a\sin(\omega t + \phi_0)$ here, $\theta = \omega t + \phi_0$ = phase of vibrating particle.

(i) Initial phase or epoch : It is the phase of a vibrating particle at $t = 0$.

In $\theta = \omega t + \phi_0$, when $t = 0$; $\theta = \phi_0$ here, ϕ_0 is the angle of epoch.

(ii) Same phase : Two vibrating particle are said to be in same phase, if the phase difference between them is an even multiple of π or path difference is an even multiple of (λ / 2) or time interval is an even multiple of (T / 2) because 1 time period is equivalent to 2π *rad* or 1 wave length (λ)

(iii) Opposite phase : When the two vibrating particles cross their respective mean positions at the same time moving in opposite directions, then the phase difference between the two vibrating particles is 180°

Opposite phase means the phase difference between the particle is an odd multiple of π (say π, 3π, 5π, 7π.....) or the path difference is an odd multiple of λ (say $\frac{\lambda}{2}, \frac{3\lambda}{2}$,.......) or the time interval is an odd multiple of (T / 2).

(iv) Phase difference : If two particles performs S.H.M and their equation are

$$y_1 = a\sin(\omega t + \phi_1) \text{ and } y_2 = a\sin(\omega t + \phi_2)$$

then phase difference $\Delta\phi = (\omega t + \phi_2) - (\omega t + \phi_1) = \phi_2 - \phi_1$

15.5 Simple Harmonic Motion

Simple harmonic motion is a special type of periodic motion, in which a particle moves to and fro repeatedly about a mean position under a restoring force which is always directed towards the mean position and whose magnitude at any instant is directly proportional to the displacement of the particle from the mean position at that instant.

Restoring force $\propto$ Displacement of the particle from mean position.

$$F \propto -x$$

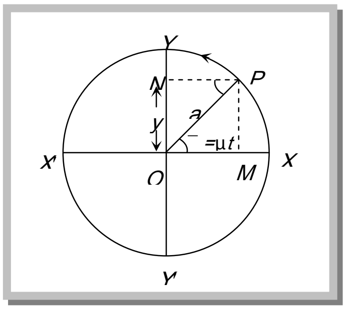
$$F = -kx$$

Where k is known as force constant. Its S.I. unit is Newton/meter and dimension is $[MT^{-2}]$.

15.6 Displacement in S.H.M.

The displacement of a particle executing S.H.M. at an instant is defined as the distance of particle from the mean position at that instant.

As we know that simple harmonic motion is defined as the projection of uniform circular motion on any diameter of circle of reference. If the projection is taken on *y*-axis.



then from the figure $y = a \sin μ\, t$

$$y = a \sin \frac{2®}{T} t$$

$$y = a \sin 2®\, n\, t$$

$$y = a \sin(μ\, t \pm ¤)$$

where *a* = Amplitude, μ = Angular frequency, *t* = Instantaneous time,

T = Time period, *n* = Frequency and ¤ = Initial phase of particle

If the projection of *P* is taken on X-axis then equations of S.H.M. can be given as

$$x = a \cos(μ\, t \pm ¤)$$

$$x = a \cos\left(\frac{2®}{T} t \pm ¤\right)$$

$$x = a \cos(2®n\, t \pm ¤)$$

Important points

(i) $y = a \sin μ\, t$ when the time is noted from the instant when the vibrating particle is at mean position.

(ii) $y = a \cos μ\, t$ when the time is noted from the instant when the vibrating particle is at extreme position.

(iii) $y = a \sin(μ\, t \pm ¤)$ when the vibrating particle is ¤ phase leading or lagging from the mean position.

(iv) Direction of displacement is always away from the equilibrium position, particle either is moving away from or is coming towards the equilibrium position.

(v) If *t* is given or phase (¯) is given, we can calculate the displacement of the particle.

If $t = \frac{T}{4}$ (or ¯ $= \frac{®}{2}$) then from equation $y = a \sin \frac{2®}{T} t$, we get $y = a \sin \frac{2®}{T} \frac{T}{4} = a \sin\left(\frac{®}{2}\right) = a$

Similarly if $t = \frac{T}{2}$ (or ¯ $= ®$) then we get $y = 0$

Sample problems based on Displacement

Problem 1. A simple harmonic oscillator has an amplitude *A* and time period *T*. The time required by it to travel from $x = A$ to $x = A/2$ is

(a) $T/6$ (b) $T/4$ (c) $T/3$ (d) $T/2$

Solution : (a) Because the S.H.M. starts from extreme position so $y = a \cos µ\, t$ form of S.H.M. should be used.

$$\frac{A}{2} = A \cos \frac{2®}{T} t \Rightarrow \cos \frac{®}{3} = \cos \frac{2®}{T} t \Rightarrow t = T/6$$

Problem 2. A mass *m* = 100 *gms* is attached at the end of a light spring which oscillates on a friction less horizontal table with an amplitude equal to 0.16 meter and the time period equal to 2 *sec*. Initially the mass is released from rest at *t* = 0 and displacement *x* = – 0.16 *meter*. The expression for the displacement of the mass at any time (*t*) is

(a) $x = 0.16 \cos (® t)$ (b) $x = -0.16 \cos(® t)$ (c) $x = 0.16 \cos(® t + ®)$ (d) $x = -0.16 \cos(® t + ®)$

Solution : (b) Standard equation for given condition

$$x = a \cos \frac{2®}{T} t \Rightarrow x = -0.16 \cos(® t)$$ [As *a* = – 0.16 *meter*, *T* = 2 *sec*]

Problem 3. The motion of a particle executing S.H.M. is given by $x = 0.01 \sin 100®(t + .05)$. Where *x* is in meter and time *t* is in seconds. The time period is

(a) 0.01 *sec* (b) 0.02 *sec* (c) 0.1 *sec* (d) 0.2 *sec*

Solution : (b) By comparing the given equation with standard equation $y = a \sin(µ\, t + ¤)$

$µ = 100®$ so $T = \frac{2®}{µ} = \frac{2®}{100®} = 0.02$ *sec*

Problem 4. Two equations of two S.H.M. are $x = a \sin(µ\, t - \sim)$ and $y = b \cos(µ\, t - \sim)$. The phase difference between the two is

(a) 0° (b) ~° (c) 90° (d) 180°

Solution : (c) $x = a \sin(µ\, t - \sim)$ and $y = b \cos(µ\, t - \sim) = b \sin(µ\, t - \sim + ®/2)$

Now the phase difference = $(\mu\ t - \sim + \frac{®}{2}) - (\mu\ t - \sim) = ® / 2 = 90^o$

15.7 Velocity in S.H.M.

Velocity of the particle executing S.H.M. at any instant, is defined as the time rate of change of its displacement at that instant.

In case of S.H.M. when motion is considered from the equilibrium position

$$y = a \sin \mu\ t$$

so $v = \frac{dy}{dt} = a\mu \cos \mu\ t$

$\therefore$ $v = a\mu \cos \mu\ t$ ……(i)

or $v = a\mu\sqrt{1 - \sin^2 \mu\ t}$ [As sinµ *t* = *y*/*a*]

or $v = \mu\sqrt{a^2 - y^2}$ …..(ii)

Important points

(i) In S.H.M. velocity is maximum at equilibrium position.

From equation (i) $v_{\max} = a\mu$ when $|\cos \mu\ t|$=1 *i.e.* ¯ = µ *t* = 0

from equation (ii) $v_{\max} = a\mu$ when $y = 0$

(ii) In S.H.M. velocity is minimum at extreme position.

From equation (i) $v_{\min} = 0$ when $|\cos \mu\ t|$ = 0 *i.e* ¯ $= \mu\ t = \frac{®}{2}$

From equation (ii) $v_{\min} = 0$ when *y* = *a*

(iii) Direction of velocity is either towards or away from mean position depending on the position of particle.

Sample problems based on Velocity

***Problem* 5.** A body is executing simple harmonic motion with an angular frequency 2 *rad/sec*. The velocity of the body at 20 *mm* displacement. When the amplitude of motion is 60 *mm* is

(a) 40 *mm/sec* (b) 60 *mm/sec* (c) 113 *mm/sec* (d) 120 *mm/sec*

Solution : (c) $v = \mu\sqrt{a^2 - y^2} = 2\sqrt{(60)^2 - (20)^2}$ = 113 *mm/sec*

Problem 6. A body executing S.H.M. has equation $y = 0.30\sin(220\,t + 0.64)$ in *meter*. Then the frequency and maximum velocity of the body is

(a) $35\,Hz, 66\,m/s$ (b) $45\,Hz, 66\,m/s$ (c) $58\,Hz, 113\,m/s$ (d) $35\,Hz, 132\,m/s$

Solution : (a) By comparing with standard equation $y = a\sin(\omega t + \phi)$ we get $a = 0.30;\ \omega = 220$

$\therefore\ 2\pi n = 220 \Rightarrow n = 35\,Hz$ so $v_{max} = a\omega = 0.3 \times 220 = 66\,m/s$

Problem 7. A particle starts S.H.M. from the mean position. Its amplitude is *A* and time period is *T*. At the time when its speed is half of the maximum speed. Its displacement *y* is

(a) *A*/2 (b) $A/\sqrt{2}$ (c) $A\sqrt{3}/2$ (d) $2A/\sqrt{3}$

Solution : (c) $v = \omega\sqrt{a^2 - y^2} \Rightarrow \frac{a\omega}{2} = \omega\sqrt{a^2 - y^2} \Rightarrow \frac{a^2}{4} = a^2 - y^2 \Rightarrow y = \frac{\sqrt{3}A}{2}$ [As $v = \frac{v_{max}}{2} = \frac{a\omega}{2}$]

Problem 8. A particle perform simple harmonic motion. The equation of its motion is $x = 5\sin(4t - \frac{\pi}{6})$. Where *x* is its displacement. If the displacement of the particle is 3 units then its velocity is

(a) $2\pi/3$ (b) $5\pi/6$ (c) 20 (d) 16

Solution : (d) $v = \omega\sqrt{a^2 - y^2} = 4\sqrt{5^2 - 3^2} = 16$ [As $\omega = 4$, $a = 5$, $y = 3$]

Problem 9. A simple pendulum performs simple harmonic motion about *x* = 0 with an amplitude (*A*) and time period (*T*). The speed of the pendulum at $x = \frac{A}{2}$ will be

(a) $\frac{\pi A\sqrt{3}}{T}$ (b) $\frac{\pi A}{T}$ (c) $\frac{\pi A\sqrt{3}}{2T}$ (d) $\frac{3\pi^2 A}{T}$

Solution : (a) $v = \omega\sqrt{a^2 - y^2} \Rightarrow v = \frac{2\pi}{T}\sqrt{A^2 - \frac{A^2}{4}} = \frac{\pi A\sqrt{3}}{T}$ [As *y* = A/2]

Problem 10. A particle is executing S.H.M. if its amplitude is 2 *m* and periodic time 2 *seconds*. Then the maximum velocity of the particle will be

(a) 6π (b) 4π (c) 2π (d) π

Solution : (c) $v_{max} = a\omega = a\frac{2\pi}{T} = 2\frac{2\pi}{2} \Rightarrow v_{max} = 2\pi$

Problem 11. A S.H.M. has amplitude '*a*' and time period *T*. The maximum velocity will be

(a) $\frac{4a}{T}$ (b) $\frac{2a}{T}$ (c) $2\pi\sqrt{\frac{a}{T}}$ (d) $\frac{2\pi a}{T}$

Solution : (d) $v_{max} = a\mu = \frac{a2®}{T}$

Problem **12.** A particle executes S.H.M. with a period of 6 second and amplitude of 3 *cm* its maximum speed in *cm/sec* is

(a) ® / 2 (b) ® (c) 2® (d) 3®

Solution : (b) $v_{max} = a\mu = a\frac{2®}{T} = 3\frac{2®}{6} \Rightarrow v_{max} = ®$

Problem **13.** A body of mass 5 *gm* is executing S.H.M. about a point with amplitude 10 *cm*. Its maximum velocity is 100 *cm/sec*. Its velocity will be 50 *cm/sec*, at a distance

(a) 5 (b) $5\sqrt{2}$ (c) $5\sqrt{3}$ (d) $10\sqrt{2}$

Solution : (c) $v_{max} = a\mu = 100\ cm/\sec$ and $a = 10\ cm$ so $\mu = 10\ rad/\sec$.

$\therefore\ v = \mu\sqrt{a^2 - y^2} \Rightarrow$ 50 = $10\sqrt{10^2 - y^2} \Rightarrow y = 5\sqrt{3}$

15.8 Acceleration in S.H.M.

The acceleration of the particle executing S.H.M. at any instant, is defined as the rate of change of its velocity at that instant. So acceleration $A = \frac{dv}{dt} = \frac{d}{dt}(a\mu\cos\mu t)$

$A = -\mu^2 a\sin\mu t$ ......(i)

$A = -\mu^2 y$ ......(ii) [As $y = a\sin\mu t$]

Important points

(i) In S.H.M. as $|\text{Acceleration}| = \mu^2 y$ is not constant. So equations of translatory motion can not be applied.

(ii) In S.H.M. acceleration is maximum at extreme position.

From equation (i) $|A_{max}| = \mu^2 a$ when $|\sin\mu t| = \text{maximum} = 1$ *i.e.* at $t = \frac{T}{4}$ or $\mu t = \frac{®}{2}$

From equation (ii) $|A_{max}| = \mu^2 a$ when $y = a$

(iii) In S.H.M. acceleration is minimum at mean position

From equation (i) $A_{min} = 0$ when $\sin\mu t = 0$ *i.e.* at $t = 0$ or $t = \frac{T}{2}$ or $\mu t = ®$

From equation (ii) $A_{min} = 0$ when $y = 0$

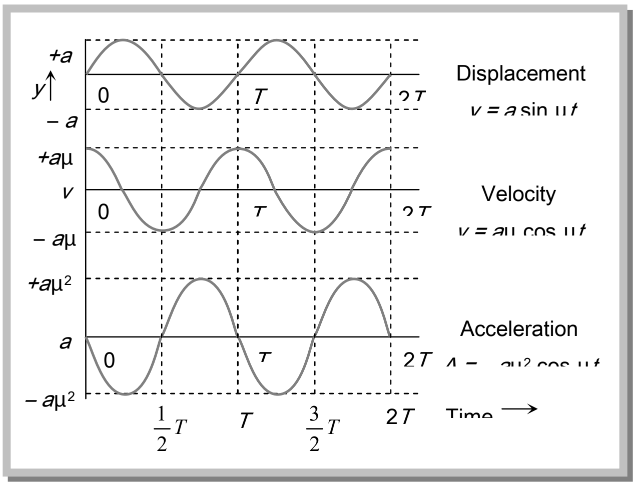
(iv) Acceleration is always directed towards the mean position and so is always opposite to displacement *i.e.,* $A \propto -y$

15.9 Comparative Study of Displacement, Velocity and Acceleration

Displacement $y = a \sin \mu\, t$

Velocity $v = a\mu \cos \mu\, t = a\mu \sin(\mu\, t + \frac{®}{2})$

Acceleration $A = -a\mu^2 \sin \mu\, t = a\mu^2 \sin(\mu\, t + ®)$



From the above equations and graphs we can conclude that.

(i) All the three quantities displacement, velocity and acceleration show harmonic variation with time having same period.

(ii) The velocity amplitude is µ times the displacement amplitude

(iii) The acceleration amplitude is μ^2 times the displacement amplitude

(iv) In S.H.M. the velocity is ahead of displacement by a phase angle ® / 2

(v) In S.H.M. the acceleration is ahead of velocity by a phase angle ® / 2

(vi) The acceleration is ahead of displacement by a phase angle of ®

(vii) Various physical quantities in S.H.M. at different position :

Physical quantities	Equilibrium position (y = 0)	Extreme Position (y = ï a)
Displacement $y = a \sin \mu\, t$	Minimum (Zero)	Maximum (a)
Velocity $v = \mu\sqrt{a^2 - y^2}$	Maximum (aµ)	Minimum (Zero)
Acceleration $\lvert A \rvert = \mu^2 y$	Minimum (Zero)	Maximum ($\mu^2 a$)

15.10 Energy in S.H.M.

A particle executing S.H.M. possesses two types of energy : Potential energy and Kinetic energy

(1) **Potential energy :** This is an account of the displacement of the particle from its mean position.

The restoring force $F = -ky$ against which work has to be done

So $$U = -\int dw = -\int_0^x F dx = \int_0^y ky\, dy = \frac{1}{2}ky^2$$

$\therefore$ potential Energy $$U = \frac{1}{2}m\mu^2 y^2 \qquad [\text{As } \mu^2 = k/m]$$

$$U = \frac{1}{2}m\mu^2 a^2 \sin^2 \mu t \qquad [\text{As } y = a\sin \mu t]$$

Important points

(i) Potential energy maximum and equal to total energy at extreme positions

$$U_{max} = \frac{1}{2}ka^2 = \frac{1}{2}m\mu^2 a^2 \qquad \text{when } y = \pm a;\ \mu t = ® / 2;\ t = T/4$$

(ii) Potential energy is minimum at mean position

$$U_{min} = 0 \qquad \text{when } y = 0;\ \mu t = 0;\ t = 0$$

(2) **Kinetic energy** : This is because of the velocity of the particle

Kinetic Energy $$K = \frac{1}{2}mv^2$$

$$K = \frac{1}{2}ma^2\mu^2 \cos^2 \mu t \qquad [\text{As } v = a\mu \cos \mu t]$$

$$K = \frac{1}{2}m\mu^2(a^2 - y^2) \qquad [\text{As } v = \mu\sqrt{a^2 - y^2}]$$

(i) Kinetic energy is maximum at mean position and equal to total energy at mean position.

$$K_{max} = \frac{1}{2}m\mu^2 a^2 \qquad \text{when } y = 0;\ t = 0;\ \mu t = 0$$

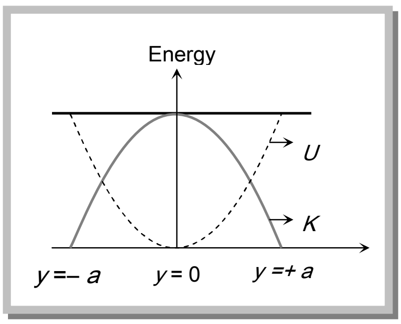
(ii) Kinetic energy is minimum at extreme position.

$$K_{min} = 0 \qquad \text{when } y = a;\ t = T/4,\ \mu t = ® / 2$$

(3) **Total energy** : Total mechanical energy = Kinetic energy + Potential energy

$$E = \frac{1}{2}m\mu^2(a^2 - y^2) + \frac{1}{2}m\mu^2 y^2 = \frac{1}{2}m\mu^2 a^2$$

Total energy is not a position function *i.e.* it always remains constant.



(4) **Energy position graph** : Kinetic energy $(K) = \frac{1}{2}m\mu^2(a^2 - y^2)$

Potential Energy (U) = $\frac{1}{2} m \mu^2 y^2$

Total Energy (E) = $\frac{1}{2} m \mu^2 a^2$

It is clear from the graph that

(i) Kinetic energy is maximum at mean position and minimum at extreme position

(ii) Potential energy is maximum at extreme position and minimum at mean position

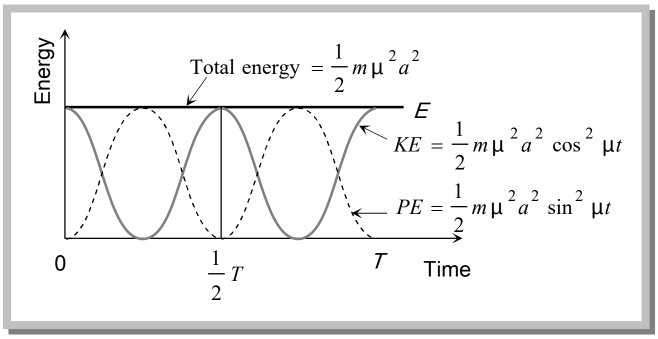
(iii) Total energy always remains constant.

(5) Kinetic Energy $K = \frac{1}{2} m \mu^2 a^2 \cos^2 \mu t = \frac{1}{4} m \mu^2 a^2 (1 + \cos 2\mu t) = \frac{1}{2} E(1 + \cos \mu' t)$

Potential Energy $U = \frac{1}{2} m \mu^2 a^2 \sin^2 \mu t = \frac{1}{4} m \mu^2 a^2 (1 - \cos 2\mu t) = \frac{1}{2} E(1 - \cos \mu' t)$

where $\mu' = 2\mu$ and $E = \frac{1}{2} m \mu^2 a^2$

i.e. in S.H.M., kinetic energy and potential energy vary periodically with double the frequency of S.H.M. (*i.e.* with time period $T' = T/2$)



From the graph we note that potential energy or kinetic energy completes two vibrations in a time during which S.H.M. completes one vibration. Thus the frequency of potential energy or kinetic energy double than that of S.H.M.

Sample problems based on Energy

Problem **14.** A particle is executing simple harmonic motion with frequency *f*. The frequency at which its kinetic energy changes into potential energy is

(a) $f/2$ (b) f (c) $2f$ (d) $4f$

Solution : (c)

Problem **15.** When the potential energy of a particle executing simple harmonic motion is one-fourth of the maximum value during the oscillation, its displacement from the equilibrium position in terms of amplitude '*a*' is

(a) $a/4$ (b) $a/3$ (c) $a/2$ (d) $2a/3$

Solution : (c) According to problem potential energy $=\frac{1}{4}$ maximum Energy

$$\Rightarrow \frac{1}{2} m \mu^2 y^2 = \frac{1}{4}\left(\frac{1}{2} m \mu^2 a^2\right) \Rightarrow y^2 = \frac{a^2}{4} \Rightarrow y = a/2$$

***Problem* 16.** A particle of mass 10 *grams* is executing S.H.M. with an amplitude of 0.5 *meter* and circular frequency of 10 *radian*/*sec*. The maximum value of the force acting on the particle during the course of oscillation is

(a) 25 *N* (b) 5 *N* (c) 2.5 *N* (d) 0.5 *N*

Solution : (d) Maximum force = mass × maximum acceleration $= m\mu^2 a = 10 \times 10^{-3}(10)^2(0.5)$ = 0.5 *N*

***Problem* 17.** A body is moving in a room with a velocity of 20 *m*/*s* perpendicular to the two walls separated by 5 meters. There is no friction and the collision with the walls are elastic. The motion of the body is

(a) Not periodic (b) Periodic but not simple harmonic

(c) Periodic and simple harmonic (d) Periodic with variable time period

Solution : (b) Since there is no friction and collision is elastic therefore no loss of energy take place and the body strike again and again with two perpendicular walls. So the motion of the ball is periodic. But here, there is no restoring force. So the characteristics of S.H.M. will not satisfied.

***Problem* 18.** Two particles executes S.H.M. of same amplitude and frequency along the same straight line. They pass one another when going in opposite directions. Each time their displacement is half of their amplitude. The phase difference between them is

(a) 30° (b) 60° (c) 90° (d) 120°

Solution : (d) Let two simple harmonic motions are $y = a\sin\mu t$ and $y = a\sin(\mu t + \text{¤})$

In the first case $\frac{a}{2} = a\sin\mu t \Rightarrow \sin\mu t = 1/2$ $\quad\therefore\quad \cos\mu t = \frac{\sqrt{3}}{2}$

In the second case $\frac{a}{2} = a\sin(\mu t + \text{¤})$

$$\Rightarrow \frac{1}{2} = [\sin\mu t.\cos\text{¤} + \cos\mu t\sin\text{¤}] \Rightarrow \frac{1}{2} = \left[\frac{1}{2}\cos\text{¤} + \frac{\sqrt{3}}{2}\sin\text{¤}\right]$$

$$\Rightarrow \quad 1 - \cos\text{¤} = \sqrt{3}\sin\text{¤} \Rightarrow (1-\cos\text{¤})^2 = 3\sin^2\text{¤} \Rightarrow (1-\cos\text{¤})^2 = 3(1-\cos^2\text{¤})$$

By solving we get $\cos\text{¤} = +1$ or $\cos\text{¤} = -1/2$

i.e. $\phi = 0$ or $\phi = 120^o$

***Problem* 19.** The acceleration of a particle performing S.H.M. is 12 *cm/sec*² at a distance of 3 *cm* from the mean position. Its time period is

(a) 0.5 *sec* (b) 1.0 *sec* (c) 2.0 *sec* (d) 3.14 *sec*

Solution : (d) $A = \omega^2 y \Rightarrow \omega = \sqrt{\frac{A}{y}} = \sqrt{\frac{12}{3}} = 2$; but $T = \frac{2\pi}{\omega} = \frac{2\pi}{2} = \pi = 3.14$

***Problem* 20.** A particle of mass 10 *gm* is describing S.H.M. along a straight line with period of 2 *sec* and amplitude of 10 *cm*. Its kinetic energy when it is at 5 cm. From its equilibrium position is

(a) $37.5\pi^2 erg$ (b) $3.75\pi^2 erg$ (c) $375\pi^2 erg$ (d) $0.375\pi^2 erg$

Solution : (c) Kinetic energy $= \frac{1}{2} m\omega^2(a^2 - y^2) = \frac{1}{2} 10 \frac{4\pi^2}{4}(10^2 - 5^2) = 375\pi^2 ergs$.

***Problem* 21.** The total energy of the body executing S.H.M. is *E*. Then the kinetic energy when the displacement is half of the amplitude is

(a) $E/2$ (b) $E/4$ (c) $3E/4$ (d) $\sqrt{3}E/4$

Solution : (c) Kinetic energy $= \frac{1}{2} m\omega^2(a^2 - y^2) = \frac{1}{2} m\omega^2\left(a^2 - \frac{a^2}{4}\right) = \frac{3}{4}\left(\frac{1}{2} m\omega^2 a^2\right) = \frac{3E}{4}$ [As $y = \frac{a}{2}$]

***Problem* 22.** A body executing simple harmonic motion has a maximum acceleration equal to 24 *m/sec*² and maximum velocity equal to 16 *meter/sec*. The amplitude of simple harmonic motion is

(a) $\frac{32}{3} meters$ (b) $\frac{3}{32} meters$ (c) $\frac{1024}{9} meters$ (d) $\frac{64}{9} meters$

Solution : (a) Maximum acceleration $\omega^2 a = 24$ …..(i)

and maximum velocity $a\omega = 16$ ….(ii)

Dividing (i) by (ii) $\omega = \frac{3}{2}$

Substituting this value in equation (ii) we get $a = 32/3 meter$

***Problem* 23.** The displacement of an oscillating particle varies with time (in seconds) according to the equation. $y(cm) = \sin\frac{\pi}{2}\left(\frac{t}{2} + \frac{1}{3}\right)$. The maximum acceleration of the particle approximately

(a) 5.21 *cm/sec*² (b) 3.62 *cm/sec*² (c) 1.81 *cm/sec*² (d) 0.62 *cm/sec*²

Solution : (d) By comparing the given equation with standard equation, $y = a\sin(\omega t + \phi)$

We find that $a = 1$ and $\omega = \pi/4$

Now maximum acceleration $= \omega^2 a = \left(\frac{\pi^2}{4}\right) = \left(\frac{3.14}{4}\right)^2 = 0.62 cm/\sec^2$

Problem 24. The potential energy of a particle executing S.H.M. at a distance *x* from the mean position is proportional to

(a) $\sqrt{x}$ (b) x (c) x^2 (d) x^3

Solution : (c)

Problem 25. The kinetic energy and potential energy of a particle executing S.H.M. will be equal, when displacement is (amplitude = *a*)

(a) $a/2$ (b) $a\sqrt{2}$ (c) $a/\sqrt{2}$ (d) $\frac{a\sqrt{2}}{3}$

Solution : (c) According to problem Kinetic energy = Potential energy $\Rightarrow \frac{1}{2}m\mu^2(a^2-y^2)=\frac{1}{2}m\mu^2y^2$

$\Rightarrow a^2-y^2=y^2 \ \therefore \ y=a/\sqrt{2}$

Problem 26. The phase of a particle executing S.H.M. is $\frac{®}{2}$ when it has

(a) Maximum velocity (b) Maximum acceleration (c) Maximum energy (d) Maximum displacement

Solution : (b, d) Phase $®/2$ means extreme position. At extreme position acceleration and displacement will be maximum.

Problem 27. The displacement of a particle moving in S.H.M. at any instant is given by $y=a\sin\mu t$. The acceleration after time $t=\frac{T}{4}$ is (where *T* is the time period)

(a) $a\mu$ (b) $-a\mu$ (c) $a\mu^2$ (d) $-a\mu^2$

Solution : (d)

Problem 28. A particle of mass *m* is hanging vertically by an ideal spring of force constant *k*, if the mass is made to oscillate vertically, its total energy is

(a) Maximum at extreme position (b) Maximum at mean position

(c) Minimum at mean position (d) Same at all position

Solution : (d)

15.11 Time Period and Frequency of S.H.M.

For S.H.M. restoring force is proportional to the displacement

$F \propto y$ or $F=-ky$ …(i) where *k* is a force constant.

For S.H.M. acceleration of the body $A=-\mu^2 y$ …(ii)

$\therefore$ Restoring force on the body $F = mA = -m\mu^2 y$ …(iii)

From (i) and (iii) $ky = m\mu^2 y \Rightarrow \mu = \sqrt{\frac{k}{m}}$

$\therefore$ Time period $(T) = \frac{2\text{®}}{\mu} = 2\text{®}\sqrt{\frac{m}{k}}$

or Frequency (n) $= \frac{1}{T} = \frac{1}{2\text{®}}\sqrt{\frac{k}{m}}$

In different types of S.H.M. the quantities m and k will go on taking different forms and names.

In general m is called inertia factor and k is called spring factor.

Thus $T = 2\text{®}\sqrt{\frac{\text{Inertia factor}}{\text{Spring factor}}}$

or $n = \frac{1}{2\text{®}}\sqrt{\frac{\text{Spring factor}}{\text{Inertia factor}}}$

In linear S.H.M. the spring factor stands for force per unit displacement and inertia factor for mass of the body executing S.H.M. and in Angular S.H.M. k stands for restoring torque per unit angular displacement and inertial factor for moment of inertia of the body executing S.H.M.

For linear S.H.M. $T = 2\text{®}\sqrt{\frac{m}{k}} = \sqrt{\frac{m}{\text{Force/Displacement}}} = 2\text{®}\sqrt{\frac{m \times \text{Displacement}}{m \times \text{Acceleration}}} = 2\text{®}\sqrt{\frac{\text{Displacement}}{\text{Acceleration}}} = 2\text{®}\sqrt{\frac{y}{A}}$

or $n = \frac{1}{2\text{®}}\sqrt{\frac{\text{Acceleration}}{\text{Dispalcement}}} = \frac{1}{2\text{®}}\sqrt{\frac{A}{y}}$

15.12 Differential Equation of S.H.M.

For S.H.M. (linear) Acceleration $\propto$ – (Displacement)

$$A \propto -y$$

or $$A = -\mu^2 y$$

or $$\frac{d^2y}{dt^2} = -\mu^2 y$$

or $$m\frac{d^2y}{dt^2} + ky = 0$$ [As $\mu = \sqrt{\frac{k}{m}}$]

For angular S.H.M. ${}^2 = -c^{-}$ and $\frac{d^{2-}}{dt^{2}} + \mu^{2-} = 0$

where $\mu^2 = \frac{c}{I}$ [As c = Restoring torque constant and I = Moment of inertia]

Sample problems based on Differential equation of S.H.M.

***Problem* 29.** A particle moves such that its acceleration a is given by $a = -bx$. Where x is the displacement from equilibrium position and b is a constant. The period of oscillation is

(a) $2® \sqrt{b}$ (b) $\frac{2®}{\sqrt{b}}$ (c) $\frac{2®}{b}$ (d) $2\sqrt{\frac{®}{b}}$

Solution : (b) We know that Acceleration = $-\mu^2$ (displacement) and $a = -bx$ (given in the problem)

Comparing above two equation $\mu^2 = b \Rightarrow \mu = \sqrt{b}$ $\therefore$ Time period $T = \frac{2®}{\mu} = \frac{2®}{\sqrt{b}}$

***Problem* 30.** The equation of motion of a particle is $\frac{d^2y}{dt^2} + ky = 0$ where k is a positive constant. The time period of the motion is given by

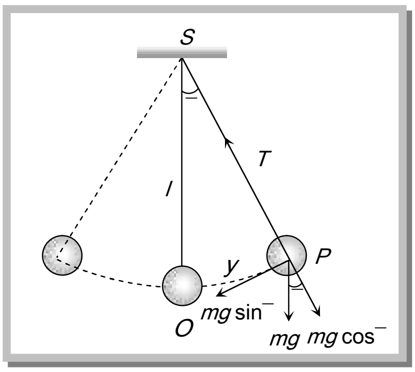
(a) $\frac{2®}{k}$ (b) $2®k$ (c) $\frac{2®}{\sqrt{k}}$ (d) $2®\sqrt{k}$

Solution : (c) Standard equation $m\frac{d^2y}{dt^2} + ky = 0$ and in a given equation m =1 and $k = k$

So, $T = 2®\sqrt{\frac{m}{k}} = \frac{2®}{\sqrt{k}}$

15.13 Simple Pendulum

An ideal simple pendulum consists of a heavy point mass body suspended by a weightless, inextensible and perfectly flexible string from a rigid support about which it is free to oscillate.



But in reality neither point mass nor weightless string exist, so we can never construct a simple pendulum strictly according to the definition.

Let mass of the bob = m

Length of simple pendulum = *l*

Displacement of mass from mean position (*OP*) = *x*

When the bob is displaced to position *P*, through a small angle θ from the vertical. Restoring force acting on the bob

$$F = -mg\sin\theta$$

or $\quad F = -mg\theta \qquad$ (When θ is small $\sin\theta \simeq \theta = \frac{\text{Arc}}{\text{Length}} = \frac{OP}{l} = \frac{x}{l}$)

or $\quad F = -mg\frac{x}{l}$

$\therefore \quad \frac{F}{x} = \frac{-mg}{l} = k$ (Spring factor)

So time period $\quad T = 2\pi\sqrt{\frac{\text{Inertia factor}}{\text{Spring factor}}} = 2\pi\sqrt{\frac{m}{mg/l}} = 2\pi\sqrt{\frac{l}{g}}$

Important points

(i) The period of simple pendulum is independent of amplitude as long as its motion is simple harmonic. But if θ is not small, $\sin\theta \neq \theta$ then motion will not remain simple harmonic but will become oscillatory. In this situation if θ_0 is the amplitude of motion. Time period

$$T = 2\pi\sqrt{\frac{l}{g}}\left[1 + \frac{1}{2^2}\sin^2\left(\frac{\theta_0}{2}\right) + ...\right] \approx T_0\left[1 + \frac{\theta_0^2}{16}\right]$$

(ii) Time period of simple pendulum is also independent of mass of the bob. This is why

(a) If the solid bob is replaced by a hollow sphere of same radius but different mass, time period remains unchanged.

(b) If a girl is swinging in a swing and another sits with her, the time period remains unchanged.

(iii) Time period $T \propto \sqrt{l}$ where *l* is the distance between point of suspension and center of mass of bob and is called effective length.

(a) When a sitting girl on a swinging swing stands up, her center of mass will go up and so *l* and hence *T* will decrease.

(b) If a hole is made at the bottom of a hollow sphere full of water and water comes out slowly through the hole and time period is recorded till the sphere is empty, initially and finally the center of mass will be at the center of the sphere. However, as water drains off the sphere, the center of mass of the system will first move down and then will come up. Due to this *l* and hence *T* first increase, reaches a maximum and then decreases till it becomes equal to its initial value.

(iv) If the length of the pendulum is comparable to the radius of earth then $T = 2® \sqrt{\frac{1}{g\left[\frac{1}{l} + \frac{1}{R}\right]}}$

(a) If $l << R$, then $\frac{1}{l} >> \frac{1}{R}$ so $T = 2® \sqrt{\frac{l}{g}}$

(b) If $l >> R (\rightarrow \infty) 1/l < 1/R$ so $T = 2® \sqrt{\frac{R}{g}} = 2® \sqrt{\frac{6.4 \times 10^6}{10}} \cong 84.6$ *minutes*

and it is the maximum time period which an oscillating simple pendulum can have

(c) If $l = R$ so $T = 2® \sqrt{\frac{R}{2g}} \cong 1 hour$

(v) If the bob of simple pendulum is suspended by a wire then effective length of pendulum will increase with the rise of temperature due to which the time period will increase.

$l = l_0 (1 + \sim \Delta¯)$ (If $\Delta¯$ is the rise in temperature, l_0 = initial length of wire, *l* = final length of wire)

$$\frac{T}{T_0} = \sqrt{\frac{l}{l_0}} = (1 + \sim \Delta¯)^{1/2} \approx 1 + \frac{1}{2} \sim \Delta¯$$

So $\frac{T}{T_0} - 1 = \frac{1}{2} \sim \Delta¯$ *i.e.* $\frac{\Delta T}{T} \approx \frac{1}{2} \sim \Delta¯$

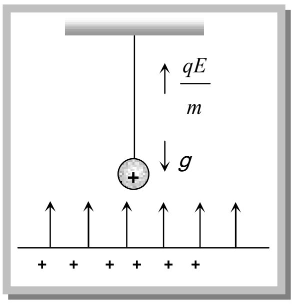
(vi) If bob a simple pendulum of density ° is made to oscillate in some fluid of density ± (where ± <°) then time period of simple pendulum gets increased.

As thrust will oppose its weight therefore $mg' = mg -$ Thrust

or $g' = g - \frac{V±g}{V°}$ *i.e.* $g' = g\left[1 - \frac{±}{°}\right] \Rightarrow \frac{g'}{g} = \frac{° - ±}{°}$

$\therefore \qquad \frac{T'}{T} = \sqrt{\frac{g}{g'}} = \sqrt{\frac{°}{° - ±}} > 1$

(vii) If a bob of mass m carries a positive charge q and pendulum is placed in a uniform electric field of strength E directed vertically upwards.



In given condition net down ward acceleration $g' = g - \frac{qE}{m}$

So $\qquad T = 2® \sqrt{\frac{l}{g - \frac{qE}{m}}}$

If the direction of field is vertically downward then time period $T = 2® \sqrt{\frac{l}{g + \frac{qE}{m}}}$

(viii) Pendulum in a lift : If the pendulum is suspended from the ceiling of the lift.

(a) If the lift is at rest or moving down ward /up ward with constant velocity.

$T = 2® \sqrt{\frac{l}{g}}$ and $n = \frac{1}{2®}\sqrt{\frac{g}{l}}$

(b) If the lift is moving up ward with constant acceleration a

$T = 2® \sqrt{\frac{l}{g + a}}$ and $n = \frac{1}{2®}\sqrt{\frac{g + a}{l}}$

Time period decreases and frequency increases

(c) If the lift is moving down ward with constant acceleration a

$T = 2® \sqrt{\frac{l}{g - a}}$ and $n = \frac{1}{2®}\sqrt{\frac{g - a}{l}}$

Time period increase and frequency decreases

(d) If the lift is moving down ward with acceleration $a = g$

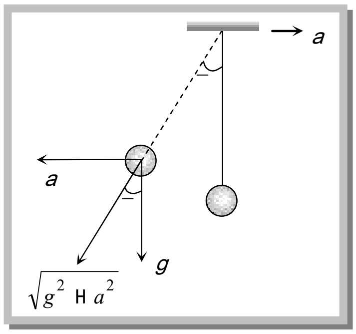
$T = 2® \sqrt{\frac{l}{g - g}} = \infty$ and $n = \frac{1}{2®}\sqrt{\frac{g - g}{l}} = 0$

It means there will be no oscillation in a pendulum.

Similar is the case in a satellite and at the centre of earth where effective acceleration becomes zero and pendulum will stop.

(ix) The time period of simple pendulum whose point of suspension moving horizontally with acceleration *a*

$$T = 2\pi\sqrt{\frac{l}{(g^2 + a^2)^{1/2}}} \quad \text{and} \quad \theta = \tan^{-1}(a/g)$$



(x) If simple pendulum suspended in a car that is moving with constant speed *v* around a circle of radius *r*.

$$T = 2\pi\frac{\sqrt{l}}{\sqrt{g^2 + \left(\frac{v^2}{r}\right)^2}}$$

(xi) Second's Pendulum : It is that simple pendulum whose time period of vibrations is two seconds.

Putting *T* = 2 *sec* and $g = 9.8 m/\sec^2$ in $T = 2\pi\sqrt{\frac{l}{g}}$ we get

$$l = \frac{4 \times 9.8}{4\pi^2} = 0.993\ m = 99.3\ cm$$

Hence length of second's pendulum is 99.3 *cm* or nearly 1 meter on earth surface.

For the moon the length of the second's pendulum will be 1/6 meter [As $g_{moon} = \frac{g_{\text{Earth}}}{6}$]

(xii) In the absence of resistive force the work done by a simple pendulum in one complete oscillation is zero.

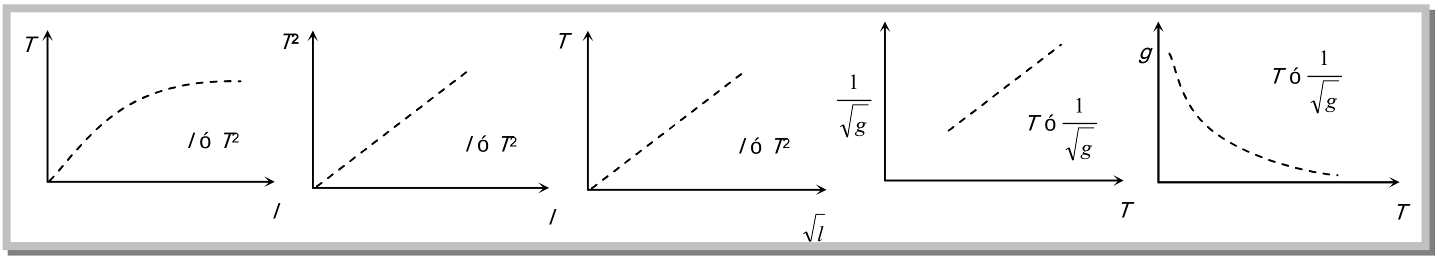
(xiii) Work done in giving an angular displacement θ to the pendulum from its mean position.

$$W = U = mgl(1 - \cos\theta)$$

(xiv) Kinetic energy of the bob at mean position = work done or potential energy at extreme

$$KE_{mean} = mgl(1 - \cos\theta)$$

(xv) Various graph for simple pendulum



Sample problems based on Simple pendulum

Problem **31.** A clock which keeps correct time at 20°*C*, is subjected to 40°*C*. If coefficient of linear expansion of the pendulum is $12\times10^{-6}/{}^\circ C$. How much will it gain or loose in time

(a) 10.3 *sec/day* (b) 20.6 *sec*/day (c) 5 *sec*/day (d) 20 *min*/day

Solution : (a) $\frac{\Delta T}{T} = \frac{1}{2}\sim\Delta^{-} = \frac{1}{2}\times 12\times10^{-6}\times(40-20)$; $\Delta T = 12\times10^{-5}\times 86400\ sec/\text{day}$ = 10.3 *sec*/day.

Problem **32.** The metallic bob of simple pendulum has the relative density °. The time period of this pendulum is *T*. If the metallic bob is immersed in water, then the new time period is given by

(a) $T\left(\frac{\circ-1}{\circ}\right)$ (b) $T\left(\frac{\circ}{\circ-1}\right)$ (c) $T\sqrt{\frac{\circ-1}{\circ}}$ (d) $T\sqrt{\frac{\circ}{\circ-1}}$

Solution : (d) Formula $\frac{T'}{T} = \sqrt{\frac{\circ}{\circ-\pm}}$ Here $\pm = 1$ for water so $T' = T\sqrt{\frac{\circ}{\circ-1}}$.

Problem **33.** The period of a simple pendulum is doubled when

(a) Its length is doubled

(b) The mass of the bob is doubled

(c) Its length is made four times

(d) The mass of the bob and the length of the pendulum are doubled

Solution : (c)

Problem **34.** A simple pendulum is executing S.H.M. with a time period *T*. if the length of the pendulum is increased by 21% the percentage increase in the time period of the pendulum is

(a) 10% (b) 21% (c) 30% (d) 50%

Solution : (a) As $T\propto\sqrt{l}$ $\therefore$ $\frac{T_2}{T_1} = \sqrt{\frac{l_2}{l_1}} = \sqrt{1.21} \Rightarrow T_2 = 1.1\,T = T + 10\%T$.

Problem **35.** The length of simple pendulum is increased by 1% its time period will

(a) Increase by 1% (b) Increase by 0.5% (c) Decrease by 0.5% (d) Increase by 2%

Solution : (b) $T = 2\pi\sqrt{l/g}$ hence $T \propto \sqrt{l}$

Percentage increment in $T = \frac{1}{2}$(percentage increment in *l*) = 0.5%.

Problem **36.** The bob of a simple pendulum of mass *m* and total energy *E* will have maximum linear momentum equal to

(a) $\sqrt{\frac{2E}{m}}$ (b) $\sqrt{2mE}$ (c) 2*mE* (d) *mE*^{2}

Solution : (b) $E = \frac{P^2}{2m}$ where *E* = Kinetic Energy, *P* = Momentum, *m* = Mass

So $P = \sqrt{2mE}$.

Problem **37.** The mass and diameter of a planet are twice those of earth. The period of oscillation of pendulum on this planet will be (if it is a second's pendulum on earth)

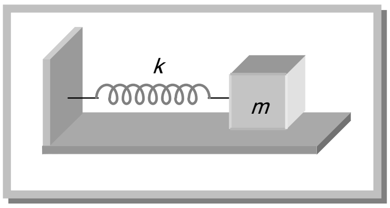
(a) $\frac{1}{\sqrt{2}}$ *sec* (b) $2\sqrt{2}$ *sec* (c) 2 *sec* (d) $\frac{1}{2}$ *sec*

Solution : (b) $g \propto \frac{M}{R^2}$; $g' = g/2$; $\frac{T'}{T} = \sqrt{\frac{g}{g'}}$ (*T* = 2 sec for second's pendulum)

$T' = 2\sqrt{2}$

15.14 Spring Pendulum

A point mass suspended from a mass less spring or placed on a frictionless horizontal plane attached with spring (fig.) constitutes a linear harmonic spring pendulum



Time period $T = 2\pi\sqrt{\frac{\text{inertia factor}}{\text{spring factor}}}$

$T = 2\pi\sqrt{\frac{m}{k}}$ and Frequency $n = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$

Important points

(i) Time period of a spring pendulum depends on the mass suspended

$T \propto \sqrt{m}$ or $n \propto \frac{1}{\sqrt{m}}$

i.e. greater the mass greater will be the inertia and so lesser will be the frequency of oscillation and greater will be the time period.

(ii) The time period depends on the force constant k of the spring

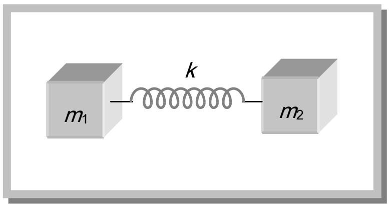
$$T \propto \frac{1}{\sqrt{k}} \qquad \text{or} \qquad n \propto \sqrt{k}$$

(iii) Time of a spring pendulum is independent of acceleration due to gravity. That is why a clock based on spring pendulum will keep proper time every where on a hill or moon or in a satellite and time period of a spring pendulum will not change inside a liquid if damping effects are neglected.

(iv) If the spring has a mass M and mass m is suspended from it, effective mass is given by $m_{eff} = m + \frac{M}{3}$

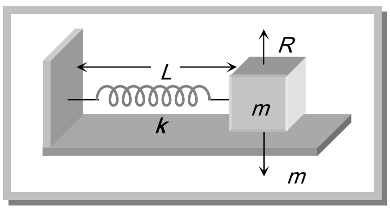
So that $$T = 2®\sqrt{\frac{m_{eff}}{k}}$$

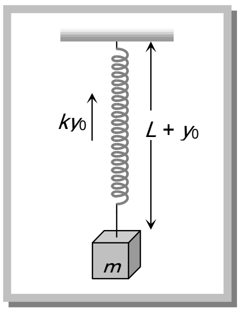
(v) If two masses of mass m_1 and m_2 are connected by a spring and made to oscillate on horizontal surface, the reduced mass m_r is given by $\frac{1}{m_r} = \frac{1}{m_1} + \frac{1}{m_2}$



So that $$T = 2®\sqrt{\frac{m_r}{k}}$$

(vi) If a spring pendulum, oscillating in a vertical plane is made to oscillate on a horizontal surface, (or on inclined plane) time period will remain unchanged. However, equilibrium position for a spring in a horizontal plain is the position of natural length of spring as weight is balanced by reaction. While in case of vertical motion equilibrium position will be $L + y_0$ with $ky_0 = mg$



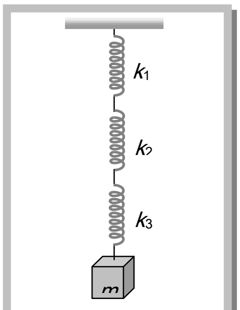


(vii) If the stretch in a vertically loaded spring is y_0 then for equilibrium of mass *m*, $ky_0 = mg$ *i.e.* $\frac{m}{k} = \frac{y_0}{g}$

So that $T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{y_0}{g}}$

Time period does not depends on '*g*' because along with *g*, y_0 will also change in such a way that $\frac{y_0}{g} = \frac{m}{k}$ remains constant

(viii) Series combination : If *n* springs of different force constant are connected in series having force constant k_1, k_2, k_3....... respectively then



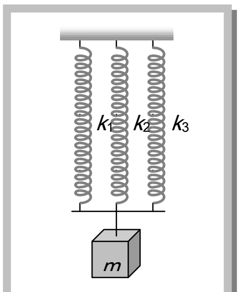
Series combination

$$\frac{1}{k_{eff}} = \frac{1}{k_1} + \frac{1}{k_2} + \frac{1}{k_3} + ...$$

If all spring have same spring constant then

$$k_{eff} = \frac{k}{n}$$

(ix) Parallel combination : If the springs are connected in parallel then



Parallel combination

$$k_{eff} = k_1 + k_2 + k_3 + ...$$

If all spring have same spring constant then

$$k_{eff} = nk$$

(x) If the spring of force constant *k* is divided in to *n* equal parts then spring constant of each part will become *nk* and if these *n* parts connected in parallel then

$$k_{eff} = n^2 k$$

(xi) The spring constant *k* is inversely proportional to the spring length.

As $k \propto \frac{1}{\text{Extension}} \propto \frac{1}{\text{Length of spring}}$

That means if the length of spring is halved then its force constant becomes double.

(xii) When a spring of length *l* is cut in two pieces of length l_1 and l_2 such that $l_1 = nl_2$.

If the constant of a spring is *k* then Spring constant of first part $k_1 = \frac{k(n+1)}{n}$

Spring constant of second part $k_2 = (n+1)k$

and ratio of spring constant $\frac{k_1}{k_2} = \frac{1}{n}$

Sample problems based on Spring pendulum

Problem 38. A spring of force constant *k* is cut into two pieces such that one pieces is double the length of the other. Then the long piece will have a force constant of

(a) $2/3k$ (b) $3/2k$ (c) $3k$ (d) $6k$

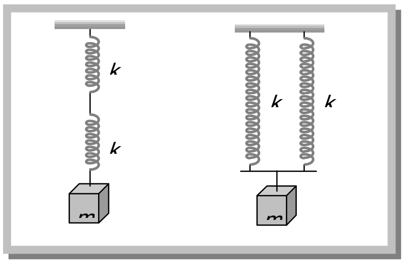
Solution : (b) If $l_1 = nl_2$ then $k_1 = \frac{(n+1)k}{n} = \frac{3}{2}k$ [As *n* = 2]

Problem 39. Two bodies *M* and *N* of equal masses are suspended from two separate mass less springs of force constants k_1 and k_2 respectively. If the two bodies oscillate vertically such that their maximum velocities are equal, the ratio of the amplitude of *M* to that of *N* is

(a) k_1/k_2 (b) $\sqrt{k_1/k_2}$ (c) k_2/k_1 (d) $\sqrt{k_2/k_1}$

Solution : (d) Given that maximum velocities are equal $a_1\mu_1 = a_2\mu_2 \Rightarrow a_1\sqrt{\frac{k_1}{m}} = a_2\sqrt{\frac{k_2}{m}} \Rightarrow \frac{a_1}{a_2} = \sqrt{\frac{k_2}{k_1}}$.

Problem 40. Two identical springs of constant *k* are connected in series and parallel as shown in figure. A mass *m* is suspended from them. The ratio of their frequencies of vertical oscillation will be



(a) 2 : 1

(b) 1 : 1

(c) 1 : 2

(d) 4 : 1

Solution : (c) For series combination $n_1 \propto \sqrt{k/2}$

For parallel combination $n_2 \propto \sqrt{2k}$ so $\frac{n_1}{n_2} = \sqrt{\frac{k/2}{2k}} = \frac{1}{2}$.

Problem 41. A block of mass *m* attached to a spring of spring constant *k* oscillates on a smooth horizontal table. The other end of the spring is fixed to a wall. The block has a speed *v* when the spring is at its natural length. Before coming to an instantaneous rest, if the block moves a distance *x* from the Mean position, then

(a) $x = \sqrt{m/k}$ (b) $x = \frac{1}{v}\sqrt{\frac{m}{k}}$ (c) $x = v\sqrt{m/k}$ (d) $x = \sqrt{mv/k}$

Solution : (c) Kinetic energy of block $\left(\frac{1}{2}mv^2\right)$ = Elastic potential energy of spring $\left(\frac{1}{2}kx^2\right)$

By solving we get $x = v\sqrt{\frac{m}{k}}$.

Problem 42. A block is placed on a friction less horizontal table. The mass of the block is *m* and springs of force constant k_1, k_2 are attached on either side with if the block is displaced a little and left to oscillate, then the angular frequency of oscillation will be

(a) $\left(\frac{k_1 + k_2}{m}\right)^{1/2}$ (b) $\left[\frac{k_1 k_2}{m(k_1 + k_2)}\right]^{1/2}$ (c) $\left[\frac{k_1 k_2}{(k_1 - k_2)m}\right]^{1/2}$ (d) $\left[\frac{k_1^2 + k_2^2}{(k_1 + k_2)m}\right]^{1/2}$

Solution : (a) Given condition match with parallel combination so $k_{eff} = k_1 + k_2 \quad \therefore\ μ = \sqrt{\frac{k_{eff}}{m}} = \sqrt{\frac{k_1 + k_2}{m}}$.

Problem 43. A particle of mass 200 *gm* executes S.H.M. The restoring force is provided by a spring of force constant 80 *N*/*m*. The time period of oscillations is

(a) 0.31 sec (b) 0.15 sec (c) 0.05 sec (d) 0.02 sec

Solution : (a) $T = 2®\sqrt{\frac{m}{k}} = 2®\sqrt{\frac{0.2}{80}} = \frac{2®}{20} = 0.31 \text{ sec}$.

Problem 44. The length of a spring is *l* and its force constant is *k* when a weight *w* is suspended from it. Its length increases by *x*. if the spring is cut into two equal parts and put in parallel and the same weight *W* is suspended from them, then the extension will be

(a) 2*x* (b) *x* (c) *x*/2 (d) *x*/4

Solution : (d) As $F = kx$ so $x \propto \frac{1}{k}$ (if *F* = constant)

If the spring of constant *k* is divided in to two equal parts then each parts will have a force constant 2*k*.

If these two parts are put in parallel then force constant of combination will becomes 4*k*.

$x \propto \frac{1}{k}$ so, $\frac{x_2}{x_1} = \frac{k_1}{k_2} = \frac{k}{4k} \Rightarrow x_2 = \frac{x}{4}$.

Problem 45. A mass *m* is suspended from a string of length *l* and force constant *k*. The frequency of vibration of the mass is f_1. The spring is cut in to two equal parts and the same mass is suspended from one of the parts. The new frequency of vibration of mass is f_2. Which of the following reaction between the frequencies is correct.

(a) $f_1 = \sqrt{2} f_2$ (b) $f_1 = f_2$ (c) $f_1 = 2f_2$ (d) $f_2 = \sqrt{2} f_1$

Solution : (d) $f \propto \sqrt{k}$

If the spring is divided in to equal parts then force constant of each part will becomes double

$\frac{f_2}{f_1} = \sqrt{\frac{k_2}{k_1}} = \sqrt{2} \Rightarrow f_2 = \sqrt{2} f_1$

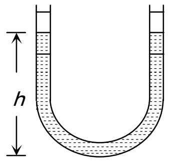
15.15 Various Formulae of S.H.M.

S.H.M. of a liquid in U tube

If a liquid of density ρ contained in a vertical U tube performs S.H.M. in its two limbs. Then time period $T = 2\pi\sqrt{\frac{L}{2g}} = 2\pi\sqrt{\frac{h}{g}}$

where L = Total length of liquid column,

h = Height of undisturbed liquid in each limb (L=2h)



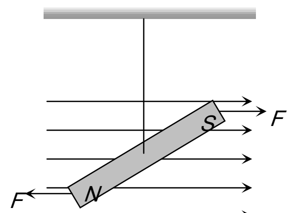
S.H.M. of a bar magnet in a magnetic field

$T = 2\pi\sqrt{\frac{I}{MB}}$

I = Moment of inertia of magnet

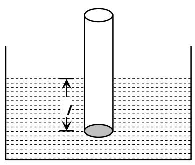
M = Magnetic moment of magnet

B = Magnetic field intensity



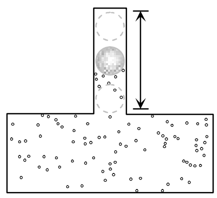
S.H.M. of a floating cylinder

If l is the length of cylinder dipping in liquid then time period $T = 2\pi\sqrt{\frac{l}{g}}$



S.H.M. of ball in the neck of an air chamber

$T = \frac{2\pi}{A}\sqrt{\frac{mV}{E}}$



m = mass of the ball

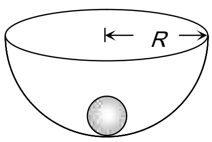
V = volume of air- chamber

A = area of cross section of neck

E = Bulk modulus for Air

S.H.M. of a small ball rolling down in hemi-spherical bowl

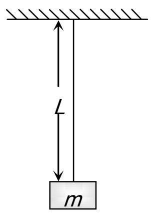
$T = 2\pi\sqrt{\frac{R-r}{g}}$



R = radius of the bowl

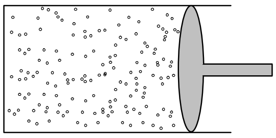
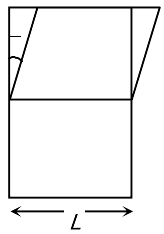
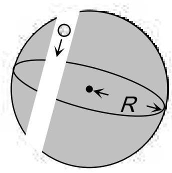
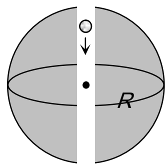
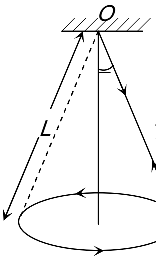
S.H.M. of a body suspended from a wire

$T = 2\pi\sqrt{\frac{mL}{YA}}$



m = mass of the body

L = length of the wire

r =radius of the ball	Y = young's modulus of wire A = area of cross section of wire
S.H.M. of a piston in a cylinder $T = 2\pi\sqrt{\frac{Mh}{PA}}$  M = mass of the piston A = area of cross section h = height of cylinder P = pressure in a cylinder	**S.H.M of a cubical block** $T = 2\pi\sqrt{\frac{M}{\eta L}}$  M = mass of the block L = length of side of cube η = modulus of rigidity
S.H.M. of a body in a tunnel dug along any chord of earth $T = 2\pi\sqrt{\frac{R}{g}}$ = 84.6 minutes  	**S.H.M. of body in the tunnel dug along the diameter of earth** $T = 2\pi\sqrt{\frac{R}{g}}$  T = 84.6 minutes R = radius of the earth = 6400km g = acceleration due to gravity = 9.8m/s^2 at earth's surface
S.H.M. of a conical pendulum $T = 2\pi\sqrt{\frac{L\cos\theta}{g}}$  L = length of string θ = angle of string from the vertical g = acceleration due to gravity	**S.H.M. of L-C circuit** $T = 2\pi\sqrt{LC}$ L = coefficient of self inductance C = capacity of condenser

15.16 Important Facts and Formulae

(1) When a body is suspended from two light springs separately. The time period of vertical oscillations are T_1 and T_2 respectively.

$$T_1 = 2\pi\sqrt{\frac{m}{k_1}} \quad \therefore \quad k_1 = \frac{4\pi^2 m}{T_1^2} \qquad \text{and} \qquad T_2 = 2\pi\sqrt{\frac{m}{k_2}} \quad \therefore \quad k_2 = \frac{4\pi^2 m}{T_2^2}$$

When these two springs are connected in series and the same mass *m* is attached at lower end and then for series combination $\frac{1}{k} = \frac{1}{k_1} + \frac{1}{k_2}$

By substituting the values of k_1, k_2 $\frac{T^2}{4®^2 m} = \frac{T_1^2}{4®^2 m} + \frac{T_2^2}{4®^2 m}$

Time period of the system $T = \sqrt{T_1^2 + T_2^2}$

When these two springs are connected in parallel and the same mass *m* is attached at lower end and then for parallel combination $k = k_1 + k_2$

By substituting the values of k_1, k_2 $\frac{4®^2 m}{T^2} = \frac{4®^2 m}{T_1^2} + \frac{4®^2 m}{T_2^2}$

Time period of the system $T = \frac{T_1 T_2}{\sqrt{T_1^2 + T_2^2}}$

(2) The pendulum clock runs slow due to increase in its time period whereas it becomes fast due to decrease in time period.

(3) If infinite spring with force constant $k, 2k, 4k, 8k$.......... respectively are connected in series. The effective force constant of the spring will be $k/2$.

(4) If $y_1 = a \sin µ\, t$ and $y_2 = b \cos µ\, t$ are two S.H.M. then by the superimposition of these two S.H.M. we get

$$\vec{y} = \vec{y}_1 + \vec{y}_2$$

$$y = a \sin µ\, t + b \cos µ\, t$$

$$y = A \sin(µ\, t + ¤)$$ this is also the equation of S.H.M.

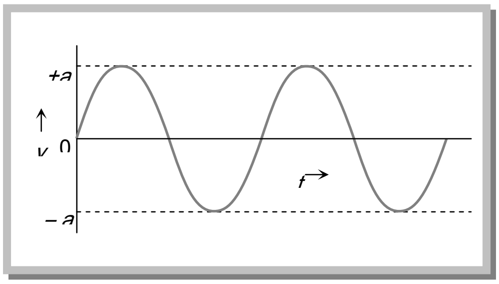
where $A = \sqrt{a^2 + b^2}$ and $¤ = \tan^{-1}(b/a)$

(5) If a particle performs S.H.M. whose velocity is v_1 at a x_1 distance from mean position and velocity v_2 at distance x_2

$$µ = \sqrt{\frac{v_1^2 - v_2^2}{x_2^2 - x_1^2}}; \quad T = 2®\sqrt{\frac{x_2^2 - x_1^2}{v_1^2 - v_2^2}} \quad a = \sqrt{\frac{v_1^2 x_2^2 - v_2^2 x_1^2}{v_1^2 - v_2^2}}; \quad v_{max} = \sqrt{\frac{v_1^2 x_2^2 - v_2^2 x_1^2}{x_2^2 - x_1^2}}$$

15.17 Free, Damped, Forced and Maintained Oscillation

(1) **Free oscillation**



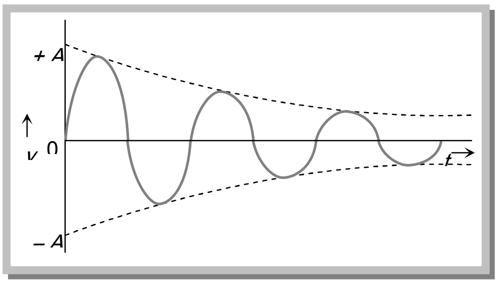
(i) The oscillation of a particle with fundamental frequency under the influence of restoring force are defined as free oscillations

(ii) The amplitude, frequency and energy of oscillation remains constant

(iii) Frequency of free oscillation is called natural frequency because it depends upon the nature and structure of the body.

(2) **Damped oscillation**

(i) The oscillation of a body whose amplitude goes on decreasing with time are defined as damped oscillation



(ii) In these oscillation the amplitude of oscillation decreases exponentially due to damping forces like frictional force, viscous force, hystersis *etc*.

(iii) Due to decrease in amplitude the energy of the oscillator also goes on decreasing exponentially

(3) **Forced oscillation**

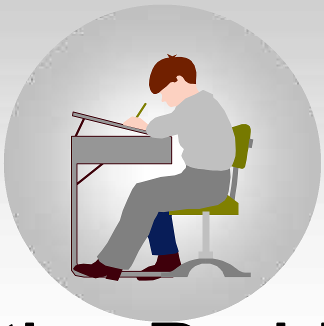
(i) The oscillation in which a body oscillates under the influence of an external periodic force are known as forced oscillation

(ii) The amplitude of oscillator decrease due to damping forces but on account of the energy gained from the external source it remains constant.

(iii) Resonance : When the frequency of external force is equal to the natural frequency of the oscillator. Then this state is known as the state of resonance. And this frequency is known as resonant frequency.

(4) **Maintained oscillation**

The oscillation in which the loss of oscillator is compensated by the supplying energy from an external source are known as maintained oscillation.



Practice Problems

Problems based on Displacement

1. A particle starts S.H.M. from the mean position. Its amplitude is *A* and time period is *T*. At the time when its speed is half of the maximum speed, its displacement *y* is

(a) $\frac{A}{2}$ (b) $\frac{A}{\sqrt{2}}$ (c) $\frac{A\sqrt{3}}{2}$ (d) $\frac{2A}{\sqrt{3}}$

2. The equation of a simple harmonic motion is $X = 0.34\cos(3000\,t + 0.74)$ where *X* and *t* are in *mm* and *s*. The frequency of motion is

(a) 3000 (b) $3000/2$® (c) $0.74/2$® (d) $3000/$®

3. A particle executes simple harmonic motion between $x = -A$ and $x = +A$. The time taken for it to go from 0 to *A*/2 is T_1 and to go from *A*/2 to *A* is T_2. Then

(a) $T_1 < T_2$ (b) $T_1 > T_2$ (c) $T_1 = T_2$ (d) $T_1 = 2T_2$

4. The equation of S.H.M. is $y = a\sin(2$®$nt + $~$)$, then its phase at time *t* is

(a) 2®*nt* (b) ~ (c) 2®*nt* + ~ (d) 2®*t*

5. A particle is executing simple harmonic motion with a period of *T seconds* and amplitude *A metre*. The shortest time it takes to reach a point $\frac{A}{\sqrt{2}}m$ from its mean position in seconds is

(a) *T* (b) *T*/4 (c) *T*/8 (d) *T*/16

6. Two particles executes S.H.M. of same amplitude and frequency along the same straight line. They pass one another when going in opposite directions, each time their displacement is half of their amplitude. The phase difference between them is

(a) 30° (b) 60° (c) 90° (d) 120°

7. A particle executing S.H.M. of amplitude 4 *cm* and *T* = 4 *sec*. The time taken by it to move from positive extreme position to half the amplitude is

(a) 1 *sec* (b) 1/3 *sec* (c) 2/3 *sec* (d) $\sqrt{3/2}$ *sec*

8. A particle is performing simple harmonic motion along *x*-axis with amplitude 4 *cm* and the period 1.2 *sec*. The minimum time taken by the particle to move from *x* = 2 *cm* to *x* = +4 *cm* and back again is given by

(a) 0.6 *s* (b) 0.4 *s* (c) 0.3 *s* (d) 0.2 *s*

9. The S.H.M. of a particle is given by the equation $y = 3 \sin \mu\, t + 4 \cos \mu\, t$. The amplitude is

(a) 7 (b) 1 (c) 5 (d) 12

10. The displacement *y* of a particle executing periodic motion is given by $y = 4 \cos^2(t/2) \sin(1000\ t)$. This expression may be considered to be a result of the superposition of …….. independent harmonic motions

(a) Two (b) Three (c) Four (d) Five

11. A S.H.M. is represented by the equation *y* = 10 sin 20®*t*. Its frequency is

(a) 10 *Hz* (b) 20 ® *Hz* (c) 0.1 *Hz* (d) 20 *Hz*

12. Equations $y_1 = A \sin \mu t$ and $y_2 = \frac{A}{2} \sin \mu t + \frac{A}{2} \cos \mu t$ represent S.H.M. The ratio of the amplitudes of the two motions is

(a) 1 (b) 2 (c) 0.5 (d) $\sqrt{2}$

13. The general equation of S.H.M. is

(a) $y = a \sin(2®t + \sim)$ (b) $y = a \sin\left(\frac{2®t}{T} + \sim\right)$ (c) $y = a \sin(\mu T + \sim)$ (d) $y = a \sin \mu t$

Problems based on Velocity

14. The velocity of a particle in simple harmonic motion at displacement *y* from mean position is

(a) $\mu\sqrt{a^2 + y^2}$ (b) $\mu\sqrt{a^2 - y^2}$ (c) μy (d) $\mu^2\sqrt{a^2 - y^2}$

15. An object is attached to the bottom of a light vertical spring and set vibrating. The maximum speed of the object is 15 *cm/sec* and the period is 628 *milli-seconds*. The amplitude of the motion in centimeters is

(a) 3.0 (b) 2.0 (c) 1.5 (d) 1.0

16. A particle in S.H.M. is described by the displacement function $x(t) = A \cos(\mu t + ¯)$. If the initial $(t = 0)$ position of the particle is 1 *cm* and its initial velocity is ® cm/s. The angular frequency of the particle is ® s^{-1}, then it's amplitude is

(a) 1 *cm* (b) $\sqrt{2}\ cm$ (c) 2 *cm* (d) 2.5 *cm*

17. If a simple pendulum oscillates with an amplitude of 50 *mm* and time period of 2 *sec*, then its maximum velocity is

(a) 0.10 *m/s* (b) 0.15 *m/s* (c) 0.8 *m/s* (d) 0.16 *m/s*

18. If a particle under S.H.M. has time period 0.1 *sec* and amplitude $2 \times 10^{-3} m$. It has maximum velocity

(a) $\frac{®}{25} m/s$ (b) $\frac{®}{26} m/s$ (c) $\frac{®}{30} m/s$ (d) None of these

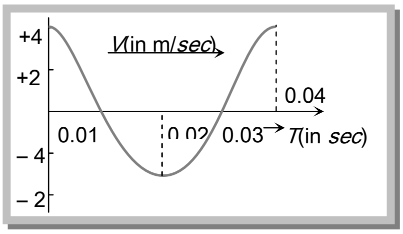
19. A particle executes simple harmonic motion with an amplitude of 4 *cm*. At the mean position the velocity of the particle is 10 *cm/s*. The distance of the particle from the mean position when its speed becomes 5 *cm/s* is

(a) $\sqrt{3}\ cm$ (b) $\sqrt{5}\ cm$ (c) $2(\sqrt{3})cm$ (d) $2(\sqrt{5})cm$

20. A body is executing S.H.M. When its displacement from the mean position is 4 *cm* and 5 *cm*, the corresponding velocity of the body is 10 *cm/sec* and 8 *cm/sec*. Then the time period of the body is

(a) 2® *sec* (b) ®/2 *sec* (c) ® *sec* (d) 3®/2 *sec*

21. The velocity-time diagram of a harmonic oscillator is shown in the adjoining figure. The frequency of oscillation is



(a) 25 *Hz*

(b) 50 *Hz*

(c) 12.25 *Hz*

(d) 33.3 *Hz*

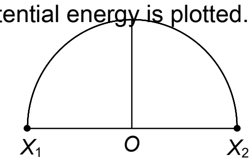
Problems based on Energy

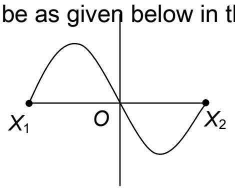
22. The total energy of a particle, executing simple harmonic motion is

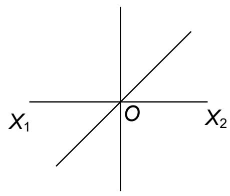
Where *x* is the displacement from the mean position

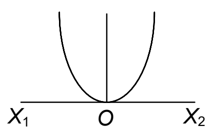
(a) $\propto x$ (b) $\propto x^2$ (c) Independent of *x* (d) $\propto x^{1/2}$

23. A particle of mass m oscillates with simple harmonic motion between points x_1 and x_2, the equilibrium position being *O*. Its potential energy is plotted. It will be as given below in the graph

(a) 

(b) 

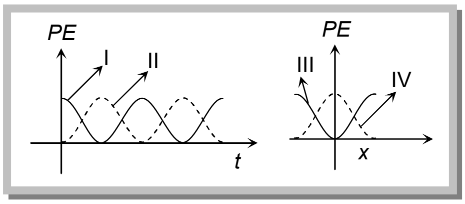
(c) 

(d) 

24. The potential energy of a simple harmonic oscillator when the particle is half way to its end point is (where *E* is the total energy)

(a) $\frac{1}{8}E$ (b) $\frac{1}{4}E$ (c) $\frac{1}{2}E$ (d) $\frac{2}{3}E$

25. For a particle executing S.H.M. the displacement *x* is given by $x = A\cos \mu t$. Identify the graph which represents the variation of potential energy (P.E.) as a function of time *t* and displacement *x*



(a) I, III

(b) II, IV

(c) II, III

(d) I, IV

26. The total energy of a particle executing S.H.M. is proportional to

(a) Displacement from equilibrium position
(b) Frequency of oscillation
(c) Velocity in equilibrium position
(d) Square of amplitude of motion

27. When the displacement is half of the amplitude, then what fraction of total energy of a simple harmonic oscillator is kinetic

(a) 3/4th
(b) 2/7th
(c) 5/7th
(d) 2/9th

28. When the displacement is half the amplitude, the ratio of potential energy to the total energy is

(a) $\frac{1}{2}$
(b) $\frac{1}{4}$
(c) 1
(d) $\frac{1}{8}$

29. The potential energy of a particle executing S.H.M. is 2.5 *J*, when its displacement is half of amplitude. The total energy of the particle be

(a) 18 *J*
(b) 10 *J*
(c) 12 *J*
(d) 2.5 *J*

30. There is a body having mass *m* and performing S.H.M. with amplitude *a.* There is a restoring force $F = -Kx$. The total energy of body depends upon

(a) *K, x*
(b) *K, a*
(c) *K, a, x*
(d) *K, a, v*

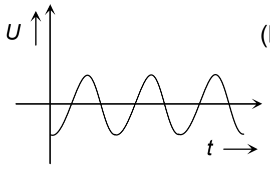
31. Two springs with spring constants $K_1 = 1500\ N/m$ and $K_2 = 3000\ N/m$ are stretched by the same force. The ratio of potential energy stored in spring will be

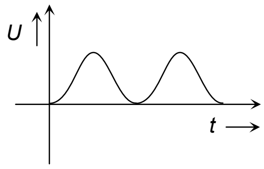
(a) 2 : 1
(b) 1 : 2
(c) 4 : 1
(d) 1 : 4

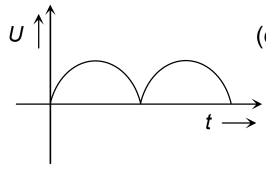
32. A body is executing Simple Harmonic Motion. At a displacement *x* its potential energy is E_1 and at a displacement *y* its potential energy is E_2. The potential energy *E* at displacement $(x + y)$ is

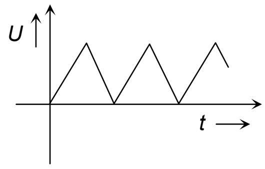
(a) $\sqrt{E} = \sqrt{E_1} - \sqrt{E_2}$ (b) $\sqrt{E} = \sqrt{E_1} + \sqrt{E_2}$ (c) $E = E_1 + E_2$ (d) $E = E_1 - E_2$

33. As a body performs S.H.M., its potential energy *U*. varies with time as indicated in

(a) 

(b) 

(c) 

(d) 

34. A particle is vibrating in a simple harmonic motion with an amplitude of 4 *cm*. At what displacement from the equilibrium position, is its energy half potential and half kinetic energy

(a) 1 *cm* (b) $\sqrt{2}$ *cm* (c) 3 *cm* (d) $2\sqrt{2}$ *cm*

35. A particle executes simple harmonic motion with a frequency *f*. The frequency with which its kinetic energy oscillates is

(a) *f*/2 (b) *f* (c) 2*f* (d) 4*f*

36. A body of mass $1\,kg$ is executing simple harmonic motion. Its displacement $y(cm)$ at *t* seconds is given by $y = 6\sin(100\,t + ®/4)$. Its maximum kinetic energy is

(a) 6 *J* (b) 18 *J* (c) 24 *J* (d) 36 *J*

37. The kinetic energy of a particle executing S.H.M. is 16 *J* when it is in its mean position. If the amplitude of oscillation is 25 *cm* and the mass of the particle is 5.12 *kg*, the time period of its oscillation is

(a) $\frac{®}{5}$ *sec* (b) 2® *sec* (c) 20® *sec* (d) 5® *sec*

Problems based on Differential equation of S.H.M.

38. If a simple harmonic oscillator has got a displacement of 0.02 *m* and acceleration equal to 2.0 ms^{-2} at any time, the angular frequency of the oscillator is equal to

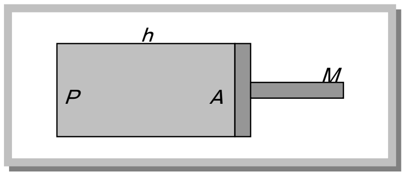
(a) 10 $rad\ s^{-1}$ (b) 0.1 $rad\ s^{-1}$ (c) 100 $rad\ s^{-1}$ (d) 1 $rad\ s^{-1}$

39. Two equal negative charges – *q* are fixed at points (0, *a*) and (0, –*a*) on the *Y*-axis. A positive charge *Q* is released from rest at point (2*a*, 0) on the *X*-axis. The charge *Q* will

(a) Execute simple harmonic motion about origin (b) Move to the origin and remained at rest

(c) Move to infinity (d) Execute oscillation but not simple harmonic motion

40. A cylindrical piston of mass M slides smoothly inside a long cylinder closed at one end, enclosing a certain mass of gas. The cylinder is kept with its axis horizontal. If the piston is disturbed from its equilibrium position, it oscillates simple harmonically. The period of oscillation will be



(a) $T = 2\pi\sqrt{\left(\frac{Mh}{PA}\right)}$

(b) $T = 2\pi\sqrt{\left(\frac{MA}{Ph}\right)}$

(c) $T = 2\pi\sqrt{\left(\frac{M}{PAh}\right)}$

(d) $T = 2\pi\sqrt{MPhA}$

41. A sphere of radius r is kept on a concave mirror of radius of curvature R. The arrangement is kept on a horizontal table (the surface of concave mirror is frictionless and sliding not rolling). If the sphere is displaced from its equilibrium position and left, then it executes S.H.M. The period of oscillation will be

(a) $2\pi\sqrt{\left(\frac{(R-r)1.4}{g}\right)}$ (b) $2\pi\sqrt{\left(\frac{R-r}{g}\right)}$ (c) $2\pi\sqrt{\left(\frac{rR}{g}\right)}$ (d) $2\pi\sqrt{\left(\frac{R}{gr}\right)}$

42. A tunnel has been dug through the centre of the earth and a ball is released in it. It will reach the other end of the tunnel after

(a) 84.6 minutes (b) 42.3 minutes (c) 1 day (d) Will not reach the other end

43. A 'U' tube of uniform bore of cross-sectional area 'a' has been set up vertically with open ends facing up. Now m *gm* of a liquid of density d is poured into it. The column of liquid in this tube will oscillate with a period T such that

(a) $T = 2\pi\sqrt{\frac{m}{g}}$ (b) $T = 2\pi\sqrt{\frac{ma}{gd}}$ (c) $T = 2\pi\sqrt{\frac{m}{gda}}$ (d) $T = 2\pi\sqrt{\frac{m}{2adg}}$

44. A block of wood has dimensions a, b and c. Its relative density is d. It is floating in water such that the side a is vertical. It is now pushed down gently and released. The time period of its S.H.M. is

(a) $T = 2\pi\sqrt{\frac{abc}{g}}$ (b) $T = 2\pi\sqrt{\frac{bc}{dg}}$ (c) $T = 2\pi\sqrt{\frac{g}{da}}$ (d) $T = 2\pi\sqrt{\frac{da}{g}}$

Problems based on Simple pendulum

45. A simple pendulum has time period T. The bob is given negative charge and surface below it is given positive charge. The new time period will be

(a) Less than T (b) Greater than T (c) Equal to T (d) Infinite

46. In a seconds pendulum, mass of the bob is 30 *gm*. If it is replaced by 90 *gm* mass. Then its time period will be

(a) 1 *sec* (b) 2 *sec* (c) 4 *sec* (d) 3 *sec*

47. The bob of a simple pendulum executes simple harmonic motion in water with a period t, while the period of oscillation of the bob is t_0 in air. Neglecting frictional force of water and given that the density of the bob is $(4/3)\times 1000\ kg/m^3$. Which relationship between t and t_0 is true

(a) $t = t_0$ (b) $t = t_0/2$ (c) $t = 2t_0$ (d) $t = 4t_0$

48. Two simple pendulum of length 5 m and 20 m respectively are given small linear displacement in one direction at the same time. They will again be in the phase when the pendulum of shorter length has completed… Oscillations

(a) 5 (b) 1 (c) 2 (d) 3

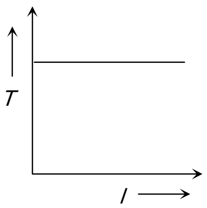
49. A chimpanzee swinging on a swing in a sitting position, stands up suddenly, the time period will

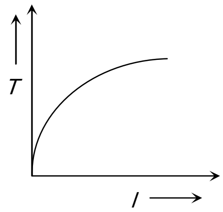
(a) Become infinite (b) Remain same (c) Increase (d) Decrease

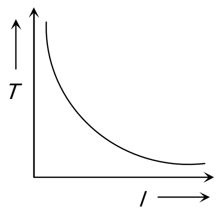
50. A simple pendulum of length l has a brass bob attached at its lower end. Its period is T. If a steel bob of same size, having density x times that of brass, replaces the brass bob and its length is changed so that period becomes 2 T, then new length is

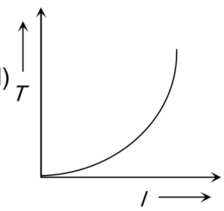
(a) 2 l (b) 4 l (c) 4 l / x (d) $\frac{4l}{x}$

51. In case of a simple pendulum, time period versus length is depicted by

(a) 

(b) 

(c) 

(d) 

52. A hollow sphere is filled with water through a small hole in it. It is then hung by a long thread and made to oscillate. As the water slowly flows out of the hole at the bottom, the period of oscillation will

(a) Continuously decrease (b) Continuously increase

(c) First decrease and then increase (d) First increase and then decrease

53. The period of oscillation of a simple pendulum of constant length at earth surface is T. Its period inside a mine is

(a) Greater than T (b) Less than T (c) Equal to T (d) Cannot be compared

54. The period of oscillation of a simple pendulum of length L suspended from the roof of a vehicle which moves without friction down an inclined plane of inclination ~, is given by

(a) 2® $\sqrt{\frac{L}{g\cos \sim}}$ (b) 2® $\sqrt{\frac{L}{g\sin \sim}}$ (c) 2® $\sqrt{\frac{L}{g}}$ (d) 2® $\sqrt{\frac{L}{g\tan \sim}}$

55. If the length of simple pendulum is increased by 300%, then the time period will be increased by

(a) 100% (b) 200% (c) 300% (d) 400%

56. If the length of second's pendulum is decreased by 2%, how many seconds it will lose per day

(a) 3927 *sec* (b) 3727 *sec* (c) 3427 *sec* (d) 864 *sec*

57. A simple pendulum consisting of a ball of mass *m* is tied to a spring of length *l* is made to swing on a circular arc of angle ¯ in a vertical plane. At the end of this arc, another ball of mass *m* is placed at rest. The momentum transferred to this ball at rest by the swinging ball is

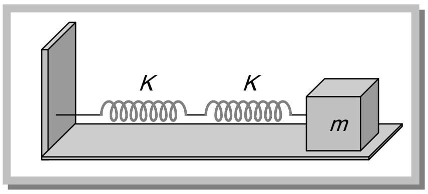
(a) Zero (b) $m¯\sqrt{\frac{g}{l}}$ (c) $\frac{m¯}{l}\sqrt{\frac{l}{g}}$ (d) $\frac{m}{l}2®\sqrt{\frac{l}{g}}$

58. A simple pendulum is suspended from the roof of a train. The train is moving with acceleration $49\ cm/\sec^2$. By what angle to the vertical, its string will be inclined

(a) 20° (b) 30° (c) Zero (d) 3° (approx.)

Problems based on Spring pendulum

59. Two springs are connected to a block of mass *M* placed on a frictionless surface as shown below. If both the springs have a spring constant *K*, the frequency of oscillation of the block is



(a) $(1/2®)\sqrt{(k/M)}$

(b) $(1/2®)\sqrt{\left(\frac{K}{2M}\right)}$

(c) $(1/2®)\sqrt{(2k/M)}$

(d) $(1/2®)\sqrt{(M/k)}$

60. Infinite springs with force constants *k*, 2*k*, 4*k* and 8*k*.... respectively are connected in series. The effective force constant of the spring will be

(a) 2*k* (b) *k* (c) *k*/2 (d) 2048

61. A spring has length *l* and spring constant *k*. if spring is divided in two equal parts then spring constant is

(a) *k* (b) *k*/2 (c) 2*k* (d) 4*k*

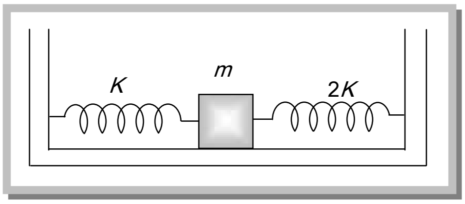
62. A particle at the end of a spring executes simple harmonic motion with a period t_1, while the corresponding period for another spring is t_2. If the period of oscillation with the two springs in series is *T*, then

(a) $T = t_1 + t_2$ (b) $T^2 = t_1^2 + t_2^2$ (c) $T^{-1} = t_1^{-1} + t_2^{-1}$ (d) $T^{-2} = t_1^{-2} + t_2^{-2}$

63. A mass M is suspended from a spring of negligible mass. The spring is pulled a little and then released so that the mass executes S.H.M. of time period T. If the mass is increased by m, the time period becomes $5T/3$. Then the ratio of m/M is

(a) $\frac{5}{3}$ (b) $\frac{3}{5}$ (c) $\frac{25}{9}$ (d) $\frac{16}{9}$

64. Two springs of force constants K and $2K$ are connected to a mass as shown below. The frequency of oscillation of the mass is



(a) (1 / 2®)$\sqrt{(K/m)}$

(b) (1 / 2®)$\sqrt{(2K/m)}$

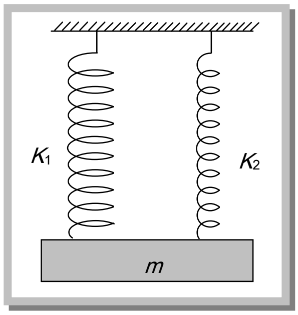
(c) (1 / 2®)$\sqrt{(3K/m)}$

(d) (1 / 2®)$\sqrt{(m/K)}$

65. One-fourth length of a spring of force constant K is cut away. The force constant of the remaining spring will be

(a) $\frac{3}{4}K$ (b) $\frac{4}{3}K$ (c) K (d) $4K$

66. A mass m is suspended separately by two different springs of spring constant K_1 and K_2 gives the time-period t_1 and t_2 respectively. If same mass m is connected by both springs as shown in figure then time-period t is given by the relation



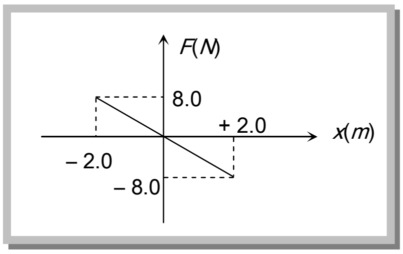
(a) $t = t_1 + t_2$

(b) $t = \frac{t_1 . t_2}{t_1 + t_2}$

(c) $t^2 = t_1^2 + t_2^2$

(d) $t^{-2} = t_1^{-2} + t_2^{-2}$

67. A body of mass 0.01 kg executes simple harmonic motion (S.H.M.) about $x = 0$ under the influence of a force shown below : The period of the S.H.M. is



(a) 1.05 s

(b) 0.52 s

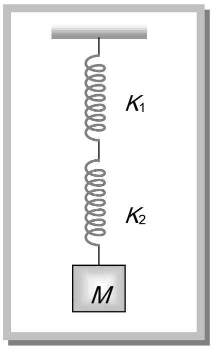
(c) 0.25 s

(d) 0.30 s

68. The force constants of two springs are K_1 and K_2. Both are stretched till their elastic energies are equal. If the stretching forces are F_1 and F_2, then $F_1 : F_2$ is

(a) $K_1 : K_2$ (b) $K_2 : K_1$ (c) $\sqrt{K_1} : \sqrt{K_2}$ (d) $K_1^2 : K_2^2$

69. A mass M is suspended by two springs of force constants K_1 and K_2 respectively as shown in the diagram. The total elongation (stretch) of the two springs is



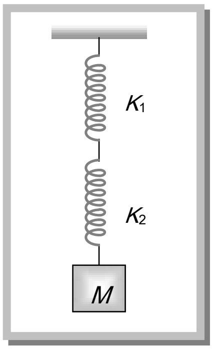
(a) $\frac{Mg}{K_1 + K_2}$

(b) $\frac{Mg(K_1 + K_2)}{K_1 K_2}$

(c) $\frac{Mg\, K_1 K_2}{K_1 + K_2}$

(d) $\frac{K_1 + K_2}{K_1 K_2 Mg}$

70. The frequency of oscillation of the springs shown in the figure will be



(a) $\frac{1}{2\text{®}}\sqrt{\frac{K}{m}}$

(b) $\frac{1}{2\text{®}}\sqrt{\frac{(K_1 + K_2)m}{K_1 K_2}}$

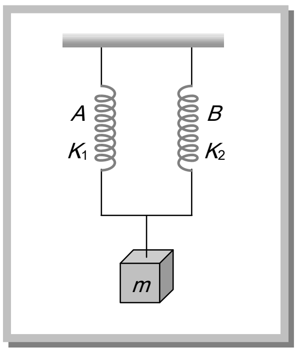
(c) $2\text{®}\sqrt{\frac{K}{m}}$

(d) $\frac{1}{2\text{®}}\sqrt{\frac{K_1 K_2}{m(K_1 + K_2)}}$

71. When a mass m is attached to a spring, it normally extends by 0.2 *m*. The mass m is given a slight additional extension and released, then its time period will be

(a) $\frac{1}{7}$ *sec* (b) 1 *sec* (c) $\frac{2\text{®}}{7}$ *sec* (d) $\frac{2}{3\text{®}}$ *sec*

72. A mass m is suspended by means of two coiled spring which have the same length in unstretched condition as in figure. Their force constant are K_1 and K_2 respectively. When set into vertical vibrations, the period will be



(a) $2\text{®}\sqrt{\frac{m}{K_1 K_2}}$

(b) $2\text{®}\sqrt{m\left(\frac{K_1}{K_2}\right)}$

(c) $2\text{®}\sqrt{\left(\frac{m}{K_1 - K_2}\right)}$

(d) $2\text{®}\sqrt{\left(\frac{m}{K_1 + K_2}\right)}$

73. A mass m attached to a spring oscillates every 2 *sec*. If the mass is increased by 2 *kg*, then time-period increases by 1 *sec*. The initial mass is

(a) 1.6 *kg* (b) 3.9 *kg* (c) 9.6 *kg* (d) 12.6 *kg*

74. A mass *M* is suspended from a light spring. An additional mass *m* added displaces the spring further by a distance *x*. Now the combined mass will oscillate on the spring with period

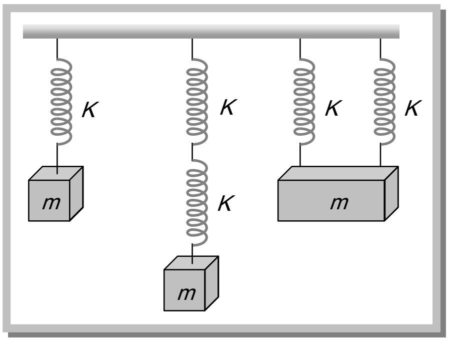
(a) $T = 2\text{®}\sqrt{[mg / x(M+m)]}$ (b) $T = 2\text{®}\sqrt{[(M+m)x / mg]}$

(c) $T = (\text{®}/2)\sqrt{[mg / x(M+m)]}$ (d) $T = 2\text{®}\sqrt{[(M+m)/mgx]}$

75. Two springs have spring constants K_A and K_B and $K_A > K_B$. The work required to stretch them by same extension will be

(a) More in spring *A* (b) More in spring *B* (c) Equal in both (d) Nothing can be said

76. Five identical springs are used in the following three configurations. The time periods of vertical oscillations in configurations (i), (ii) and (iii) are in the ratio



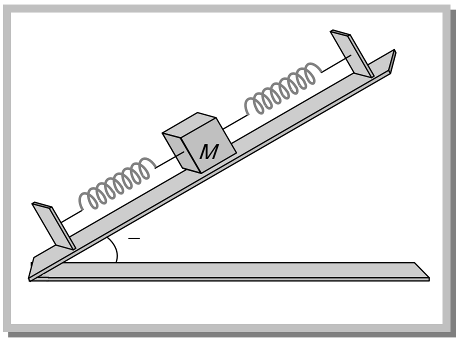
(a) $1 : \sqrt{2} : \frac{1}{\sqrt{2}}$

(b) $2 : \sqrt{2} : \frac{1}{\sqrt{2}}$

(c) $\frac{1}{\sqrt{2}} : 2 : 1$

(d) $2 : \frac{1}{\sqrt{2}} : 1$

77. On a smooth inclined plane, a body of mass *M* is attached between two springs. The other ends of the springs are fixed to firm supports. If each spring has force constant *K*, the period of oscillation of the body (assuming the spring as massless) is



(a) $2\text{®}\left(\frac{M}{2K}\right)^{1/2}$

(b) $2\text{®}\left(\frac{2M}{K}\right)^{1/2}$

(c) $2\text{®}\,\frac{Mg\sin\text{¯}}{2k}$

(d) $2\text{®}\left(\frac{2Mg}{K}\right)^{1/2}$

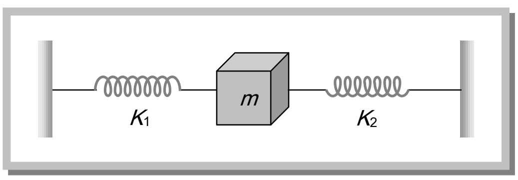
78. The length of a spring is *l* and its force constant is *k*. When a weight *W* is suspended from it, its length increases by *x*. if the spring is cut into two equal parts and put in parallel and the same weight *W* is suspended from them, then the extension will be

(a) 2*x* (b) *x* (c) $\frac{x}{2}$ (d) $\frac{x}{4}$

79. In arrangement given in figure, if the block of mass *m* is displaced, the frequency is given by

(a) $n = \frac{1}{2\text{®}}\sqrt{\left(\frac{K_1 - K_2}{m}\right)}$

(b) $n = \frac{1}{2\text{®}}\sqrt{\left(\frac{K_1 + K_2}{m}\right)}$



(c) $n = \frac{1}{2\pi}\sqrt{\left(\frac{m}{K_1 + K_2}\right)}$

(d) $n = \frac{1}{2\pi}\sqrt{\left(\frac{m}{K_1 - K_2}\right)}$

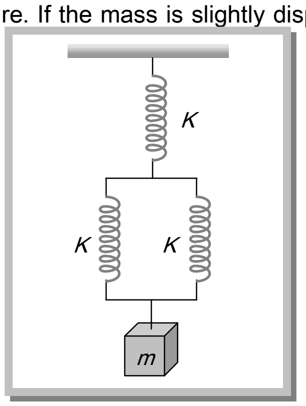
80. A mass *M* is suspended from a spring of negligible mass. The spring is pulled a little and then released so that the mass executes simple harmonic oscillations with a time period *T*. If the mass is increased by *m*, then the time period becomes $\left(\frac{5}{4}T\right)$. The ratio of *m/M* is

(a) 9/16 (b) 25/16 (c) 4/5 (d) 5/4

81. A mass on the end of a spring undergoes simple harmonic motion with a frequency of 0.5 *Hz*. If the attached mass is reduced to one quarter of its value, then the new frequency in *Hz* is

(a) 0.25 (b) 1.0 (c) 2.0 (d) 4.5

82. A body of mass *m* hangs from three springs, each of spring constant *K* as shown in the figure. If the mass is slightly displaced and let go, the system will oscillate with time period



(a) $2\pi\sqrt{\frac{m}{3K}}$

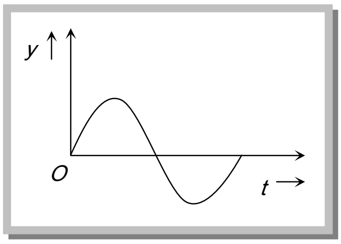
(b) $2\pi\sqrt{\frac{3m}{2K}}$

(c) $2\pi\sqrt{\frac{2m}{3K}}$

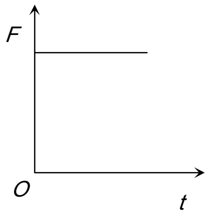
(d) $2\pi\sqrt{\frac{3K}{m}}$

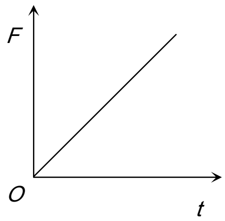
Miscellaneous Problems

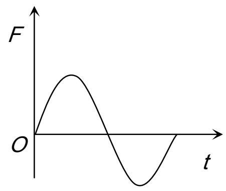
83. The displacement time graph of a particle executing S.H.M. is as shown in the figure

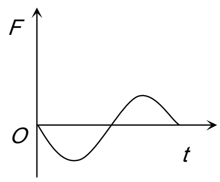


The corresponding force-time graph of the particle is

(a) 

(b) 

(c) 

(d) 

84. The amplitude of a damped oscillator becomes half on one minute. The amplitude after 3 minute will be $\frac{1}{X}$ times the original, where *X* is

(a) 2×3 (b) 2^3 (c) 3^2 (d) 3×2^2

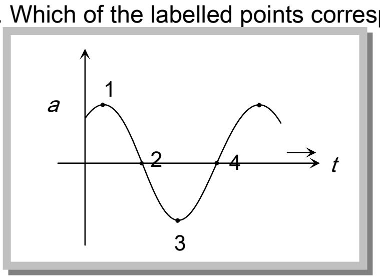
85. The maximum speed of a particle executing S.H.M. is 1 *m/s* and its maximum acceleration is 1.57 *m/sec*^{2}. The time period of the particle will be

(a) $\frac{1}{1.57}$ *sec* (b) 1.57 *sec* (c) 2 *sec* (d) 4 *sec*

86. The amplitude of a particle executing S.H.M. with frequency of 60 *Hz* is 0.01 *m*. The maximum value of the acceleration of the particle is

(a) $144®^2\ m/\sec^2$ (b) $144\ m/\sec^2$ (c) $\frac{144}{®^2} m/\sec^2$ (d) $288®^2\ m/\sec^2$

87. The acceleration *a* of a particle undergoing S.H.M. is shown in the figure. Which of the labelled points corresponds to the particle being at – x_{max}



(a) 4

(b) 3

(c) 2

(d) 1

88. A 0.10 *kg* block oscillates back and forth along a horizontal surface. Its displacement from the origin is given by: $x = (10\,cm)\cos[(10\,rad/s)t + ®/2\,rad]$. What is the maximum acceleration experienced by the block

(a) $10\ m/s^2$ (b) $10®\ m/s^2$ (c) $\frac{10®}{2} m/s^2$ (d) $\frac{10®}{3} m/s^2$

89. A simple pendulum is set into vibrations. The bob of the pendulum comes to rest after sometime due to

(a) Air friction (b) Moment of inertia (c) Weight of the bob (d) Combination of all the above

90. A particle of mass 10 *grams* is executing simple harmonic motion with amplitude of 0.5 *m* and periodic time of (®/5) *seconds*. The maximum value of the force acting on the particle is

(a) 25 *N* (b) 5 *N* (c) 2.5 *N* (d) 0.5 *N*

91. The acceleration of a particle performing S.H.M. is 12 *cm/sec*² at a distance of 3 *cm* from the mean position. Its time period is

(a) 0.5 *sec* (b) 1.0 *sec* (c) 2.0 *sec* (d) 3.14 *sec*

92. A large horizontal surface moves up and down in S.H.M. with an amplitude of 1 *cm*. If a mass of 10 *kg* (which is placed on the surface) is to remain continually in contact with it, the maximum frequency of S.H.M. will be

(a) 0.5 *Hz* (b) 1.5 *Hz* (c) 5 *Hz* (d) 10 *Hz*

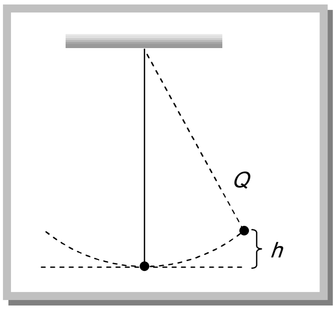
93. The bob of a simple pendulum is displaced from its equilibrium position *O* to a position *Q* which is at height *h* above *O* and the bob is then released. Assuming the mass of the bob to be *m* and time period of oscillations to be 2.0 *sec*, the tension in the string when the bob passes through *O* is

(a) $m(g + ®\sqrt{2gh})$

(b) $m(g + \sqrt{®^2 gh})$

(c) $m\left(g + \sqrt{\frac{®^2}{2} gh}\right)$

(d) $m\left(g + \sqrt{\frac{®^2}{3} gh}\right)$

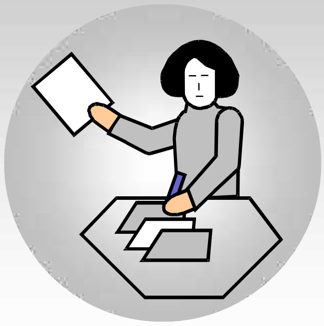


94. One end of a long metallic wire of length *L* is tied to the ceiling. The other end is tied to massless spring of spring constant *K*. a mass *m* hangs freely from the free end of the spring. The area of cross-section and Young's modulus of the wire are *A* and *Y* respectively. If the mass is slightly pulled down and released, it will oscillate with a time period *T* equal to

(a) $2®\left(\frac{m}{K}\right)$ (b) $2®\left\{\frac{(YA + KL)m}{YAK}\right\}^{1/2}$ (c) $2®\frac{mYA}{KL}$ (d) $2®\frac{mL}{YA}$

95. The amplitude of vibration of a particle is given by $a_m = (a_0)/(aμ^2 - bμ + c)$; where a_0, *a*, *b* and *c* are positive. The condition for a single resonant frequency is

(a) $b^2 < 4ac$ (b) $b^2 > 4ac$ (c) $b^2 = 5ac$ (d) $b^2 = 7ac$



Answer Sheet (Practice problems)

1.	2.	3.	4.	5.	6.	7.	8.	9.	10.
c	b	a	c	c	d	c	b	c	b
11.	12.	13.	14.	15.	16.	17.	18.	19.	20.
a	d	b	b	c	b	b	a	c	c
21.	22.	23.	24.	25.	26.	27.	28.	29.	30.
a	c	d	b	a	d	a	b	b	b
31.	32.	33.	34.	35.	36.	37.	38.	39.	40.
a	b	b	d	c	b	a	a	d	a
41.	42.	43.	44.	45.	46.	47.	48.	49.	50.
b	b	d	d	a	b	c	c	d	b
51.	52.	53.	54.	55.	56.	57.	58.	59.	60.
b	d	a	a	a	d	a	d	c	d
61.	62.	63.	64.	65.	66.	67.	68.	69.	70.
c	b	d	c	b	d	d	c	b	d
71.	72.	73.	74.	75.	76.	77.	78.	79.	80.
c	d	a	b	a	a	a	d	b	a
81.	82.	83.	84.	85.	86.	87.	88.	89.	90.
b	b	d	b	d	a	d	a	a	d
91.	92.	93.	94.	95.					
d	c	a	b	a					