LESSON 3

MATRICES

1. DEFINITION OF MATRIX

A system of '*mn*' numbers (real or complex) arranged in a rectangular array of *m* rows and *n* columns is called a matrix. This system is arranged in any of the following patterns.

$$\left(\quad\right);\ \left[\quad\right];\ \left\|\quad\right\|$$

In general a_{ij} represent the element (or entry) of i^{th} row and j^{th} column, so the matrix can be represented as (a_{ij}) or $[a_{ij}]$ or $||a_{ij}||$

2. ORDER OF MATRIX

If any matrix A contains '*m*' rows and '*n*' columns then $m \times n$ is termed as order of matrix.

Order is generally written as suffix of the array.

Now any matrix of order $m \times n$ will have the notation $[a_{ij}]_{m\times n}$.

i.e. $A = [a_{ij}]_{m\times n}$ or $(a_{ij})_{m\times n}$ or $\|a_{ij}\|_{m\times n}$

it is obvious that $1 \le i \le m$ and $1 \le j \le n$

Illustration 1

Question: **In the inter sports meet of local colleges the games to be played are T.T., Hockey, Badminton, Tennis, and B. Ball. The three colleges of Meerut sent the following number of players.**

Meerut College (*M*.C.) J **35 players ; 5(T. T.), 11 (Hockey), 5(Bad), 6 (Tennis) and 8(B. Ball).**

Nanak Chand College (N. A. S.) J **22 players ; 3(T. T), 13 (Hockey), 2 (Bad), 4 (Tennis) and none for (B. Ball).**

Dev Nagri College (D. N.) J **31 players ; 2(T. T.), 15 (Hockey) 3(Bad), 5 (Tennis) and 6 (B. Ball). Put this information in matrix form.**

Solution: The above information can be put in tabular form as under.

Colleges	Number of players				
	T.T.	Hockey	Badminton	Tennis	B. Ball
M. C. (35)	5	11	5	6	8
N. A. S. (22)	3	13	2	4	0
D. *N*. (31)	2	15	3	5	6

The number 4 represents the number of players the N.A.S. College has sent for playing Tennis. The number 15 represents the number of players the D.N. college has sent for playing Hockey. Similarly, the numbers 8 represents the number of players which Meerut College has sent for playing basket ball. The above can be put in rectangular array form as

$$\begin{bmatrix} 5 & 11 & 5 & 6 & 8 \\ 3 & 13 & 2 & 4 & 0 \\ 2 & 15 & 3 & 5 & 6 \end{bmatrix}$$

Above is a 3×5 matrix, 3 represents the number of rows (number of colleges) participating and 5 represents the number of games being played in the meet.

3. TYPES OF MATRIX DEPENDING UPON THEIR ENTRIES

The elements which appear in the rectangular array are known as entries ; depending upon these entries, matrices are of following types:

3.1 ROW MATRIX

A single row matrix is called a row matrix or a row vector.

e.g. the matrix $[a_{11}\ a_{12}\ \ldots..\ \ a_{1n}]$ is a $1 \times n$ row matrix.

3.2 COLUMN MATRIX

A single column matrix is called a column matrix or a column vector.

e.g. the matrix $\begin{bmatrix} a_{1j} \\ a_{2j} \\ \ldots \\ \ldots \\ a_{mj} \end{bmatrix}$ is a $m \times 1$ column matrix.

3.3 SQUARE MATRIX

If $m = n$, i.e. if the number of rows and columns of a matrix are equal say n, then it is called a square matrix of order n.

3.4 NULL (or zero) MATRIX

If all the elements of a matrix are equal to zero, then it is called a null matrix and is denoted by $O_{m \times n}$ or O.

3.5 DIAGONAL MATRIX

A square matrix in which all its elements are zero except those in the leading diagonal is called a diagonal matrix. Thus in a diagonal matrix $a_{ij} = 0$ if $i \neq j$.

The diagonal matrices of order 2 and 3 are as follows:

$$\begin{bmatrix} k_1 & 0 \\ 0 & k_2 \end{bmatrix}, \begin{bmatrix} k_1 & 0 & 0 \\ 0 & k_2 & 0 \\ 0 & 0 & k_3 \end{bmatrix}.$$

The elements a_{ij} of a matrix for which i = j are called the diagonal elements of a matrix and the diagonal along which all these elements lie is called the principal diagonal or the diagonal of the matrix.

3.6 SCALAR MATRIX

A square matrix in which all the diagonal elements are equal and all other elements equal to zero is called a scalar matrix.

i.e. in a scalar matrix $a_{ij} = k$, for i = j and $a_{ij} = 0$ for i $\neq j$. Thus $\begin{bmatrix} k & 0 & 0 \\ 0 & k & 0 \\ 0 & 0 & k \end{bmatrix}$ is a scalar matrix.

3.7 UNIT MATRIX OR IDENTITY MATRIX

A square matrix in which all its diagonal elements are equal to 1 and all other elements equal to zero is called a unit matrix or identity matrix.

e.g. a unit (or identity) matrix of order 2 and 3 are $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$.

3.8 NEGATIVE OF A MATRIX

Let A = $[a_{ij}]_{m\times n}$ be a matrix. Then the negative of the matrix A is defined as the matrix $[-a_{ij}]_{m\times n}$ and is denoted by –A.

4. EQUALITY OF MATRICES

Two matrices *A* and *B* are said to be equal, written as *A* = *B*, if,

(i) they are both of the same order i.e. have the same number of rows and columns, and

(ii) the elements in the corresponding places of the two matrices are the same.

5. ADDITON AND SUBTRACTION OF MATRICES

Let $A = [a_{ij}]$ and $B = [b_{ij}]$ be two matrices of the same type $m \times n$. Then their sum (or difference) $A + B$ (or $A - B$) is defined as another matrix of the same type, say $C = [c_{ij}]$ such that any element of *C* is the sum (or difference) of the corresponding elements of *A* and *B*.

$\therefore \quad C = A \pm B = [a_{ij} \pm b_{ij}]$

Illustration 2

Question: $A = \begin{bmatrix} 1 & 2 & 4 \\ 0 & 5 & 3 \end{bmatrix}$ **and B =** $\begin{bmatrix} 7 & 3 & 2 \\ 5 & 1 & 9 \end{bmatrix}$.

Solution: Here both *A* and *B* are 2 × 3 matrices

$$\therefore \quad A + B = \begin{bmatrix} 1+7 & 2+3 & 4+2 \\ 0+5 & 5+1 & 3+9 \end{bmatrix} = \begin{bmatrix} 8 & 5 & 6 \\ 5 & 6 & 12 \end{bmatrix}$$

$$\text{and} \quad A - B = \begin{bmatrix} 1-7 & 2-3 & 4-2 \\ 0-5 & 5-1 & 3-9 \end{bmatrix} = \begin{bmatrix} -6 & -1 & 2 \\ -5 & 4 & -6 \end{bmatrix}$$

5.1 PROPERTIES OF MATRIX ADDITION

1. $A + B = B + A$

2. $A + (B + C) = (A + B) + C$

3. $k(A + B) = kA + kB$ here *k* is any scalar.

4. *A* + O = O + *A* = *A* , here O {null matrix} will be additive identity.

5. If A be a given matrix then the matrix $-A$ is the additive inverse of A for $A + (-A)$ = null matrix O.

6. If A, B and C be three matrices of the same type
then $A + B = A + C \Rightarrow B = C$ (L. C. Law)
and $B + A = C + A \Rightarrow B = C$ (R. C. Law)
where L means left and R means right.

6. MULTIPLICATION OF MATRIX BY A SCALAR

Let $A = [a_{ij}]_{m\times n}$ be a matrix and k a scalar. Then the matrix obtained by multiplying each element of matrix A by k is called the scalar multiple of A by k and is denoted by $k\,A$ or $A\,k$.

6.1 PROPERTIES

- If k_1 and k_2 are scalars and A be a matrix, then $(k_1 + k_2)\,A = k_1A + k_2A$.
- If k_1 and k_2 are scalars and A be a matrix, then $k_1(k_2\,A) = (k_1k_2)A$.
- If A and B are two matrices of the same order and k, a scalar, then $k(A + B) = kA + kB$.
  i.e. the scalar multiplication of matrices distributes over the addition of matrices.
- If k_1 and k_2 are two scalars and A is any matrix, then $(k_1 + k_2)\,A = k_1A + k_2A$.
- If A is any matrix and k be a scalar, then $(-k)A = -(kA) = k(-A)$.

7. MULTIPLICATION OF TWO MATRICES

Let $A = [a_{ij}]$ be $m \times p$ matrix and $B = [b_{ij}]$ be $p \times n$ matrix. These matrices A and B are such that the number of columns of A are the same as the number of rows of B each being equal to p. Then the product AB (in the order it is written) will be a matrix $C = [c_{ij}]$ of the type $m \times n$.

Where c_{ij} will be the element of C occurring in i^{th} row and j^{th} column and it will be row by column product of i^{th} row of A having p columns with j^{th} column of B having p rows, the elements of which are

$a_{i1}\ a_{i2}\ \ldots..\ a_{ip}$ and b_{1j}

$a_{i1}\ a_{i2}\ \ldots..\ a_{ip}$ and b_{2j}

………………………

………………………

$$\therefore \quad c_{ij} = a_{i1}\,b_{1j} + a_{i2}\,b_{2j} + \ldots..\ a_{ip}\,b_{pj} = \sum_{k=1}^{p} a_{ik}b_{kj}$$

The summation is to be performed w.*r*.t. repeated suffix k.

Above gives us the particular i-jth element of C which is $m \times n$ type. For getting an element of C occurring in 2nd row and 3rd column we shall put $i = 2$ and $j = 3$.

$\therefore \quad c_{23} = \sum_{k=1}^{p} a_{2k} b_{k3} = a_{21}b_{13} + a_{22}b_{23} + ...a_{2p}b_{p3}$

There being *m* rows in *A*, *i* can take values from 1 to *m* and there being *n* columns in *B*, *j* can take values from 1 to *n*, and thus we shall get all the *mn* elements of *C*.

Again $\quad c_{ij} = \sum_{k=1}^{p} a_{ik} b_{kj}$ …(i)

Above gives us *i-j*th element of *AB* which is of $m \times n$ type having *m* rows and *n* columns.

7.1 ELEMENTS OF *j*th COLUMN OF AB

For getting elements of *j*th column, *j* will remain fixed for *j*th column whereas *i* will change from 1 to *m* as there are *m* rows in AB.

Hence giving *i* the values 1, 2, 3……*m* and keeping *j* fixed in (i) we shall get all the elements of *j*th column of AB.

$\therefore$ *j*th column of AB is $\sum_{k=1}^{p} a_{1k}\ b_{kj}\ , \sum_{k=1}^{p} a_{2k} b_{kj}\ , ... , \sum_{k=1}^{p} a_{mk} b_{kj}$

7.2 AN EASY WAY TO REMEMBER

If we denote the ordered set of rows of *A* by R_1, R_2, R_3 each having 2 elements and ordered set of columns of *B* by C_1, C_2, each having 2 elements.

then $AB = \begin{bmatrix} R_1 \\ R_2 \\ R_3 \end{bmatrix}_{3\times1} [C_1 C_2]_{1\times2} = \begin{bmatrix} R_1C_1 & R_1C_2 \\ R_2C_1 & R_2C_2 \\ R_3C_1 & R_3C_2 \end{bmatrix}_{3\times2}$

7.3 FEW IMPORTANT THINGS FOR THE MULTIPLICATION

1. **Condition for product AB to exist or to be defined:** If *A* and *B* be two matrices then their product is defined or in other words *A* is *conformable* to *B* for multiplication if the number of columns of *A* is the same as the number of rows in *B*. i.e. If *A* be $m \times p$ and *B* be $p \times n$, the matrix *AB* will be of the type $m \times n$.

2. **Pre-multiplication and post multiplication**

   When we say multiply *A* by *B* then it could mean both *AB* or *BA* where *A* and *B* are any numbers. But when *A* and *B* are matrices then as seen above *AB* and BA do not necessarily mean the same thing. If *AB* is defined for matrix multiplication *BA* may not be defined. To avoid this when we say product *AB* it would mean the matrix *A* post-multiplied by *B* and when we say product *BA* it would mean matrix *A* pre-multiplied by *B*. In *AB*, *A* is called *prefactor* and *B* *post factor*.

3. In the case when both *A* and *B* are square matrices of the same type then also both *AB* and *BA* are defined and the product matrix is also a matrix of the same type but still $AB \neq BA$.

4. Again we know that when $ab = 0$ it means that either *a* or *b* (or both) is zero. But $AB = O$ i.e. a null matrix does not necessarily imply that either *A* or $B = O$ as shown above because neither *A* nor *B* is null matrix whereas *AB* is a null matrix.

Illustration 3

Question: **If $A = \begin{bmatrix} 1 & -1 & 1 \\ -3 & 2 & -1 \\ -2 & 1 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 1 & 2 & 3 \end{bmatrix}$ compute AB and BA.**

Solution: Here A is 3×3 and B is 3×3. Hence both AB and BA are defined and each will be 3×3 matrix.

Let $AB = C = \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix}$

where C_{ij} means that take the product of ith row of A with jth column of B.

e.g. C_{23} = product of 2nd row of A with 3rd column of B.

i.e. $[-3 \quad 2 \quad -1] \begin{bmatrix} 3 \\ 6 \\ 3 \end{bmatrix} = -3.3 + 2.6 - 1.3 = 0$.

Similarly we can find other elements of C.

We can also say that by the product of first row of A with the three columns of B ; we shalll get the three elements of first row of C.

i.e. R_1C_1, R_1C_2, R_1C_3

and similarly take the second row of A and multiply with all the columns of B and you will get the three elements of 2nd row of C i.e. R_2C_1, R_2C_2, R_2C_3 and elements of 3rd row of C will be R_3C_1, R_3C_2, R_3C_3.

$$\therefore \quad AB = \begin{bmatrix} 1.1 - 1.2 + 1.1 & 1.2 - 1.4 + 1.2 & 1.3 - 1.6 + 1.3 \\ -3.1 + 2.2 - 1.1 & -3.2 + 2.4 - 1.2 & -3.3 + 2.6 - 1.3 \\ -2.1 + 1.2 + 0.1 & -2.2 + 1.4 + 0.2 & -2.3 + 1.6 + 0.3 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = O \text{ (i.e. , null matrix).}$$

Illustration 4

Question: **If *A* and *B* be matrices such that both, *AB* and *A* + *B* are defined. Prove that both *A* and *B* are square matrices of the same order.**

Solution: We know that two matrices A and B are conformable for addition if they are of the same type. Thus if A be $m \times n$ then B should also be $m \times n$ as $A + B$ is defined.

Again since AB is also defined therefore number of columns in A i.e., n should be equal to number of rows in B i.e. m. Hence $n = m$ and in that case both A and B will be square matrices of order equal to $m = n$.

Illustration 5

Question: **If A be any $m \times n$ matrix and both AB and BA are defined prove that B should be $n \times m$ matrix.**

Solution: Since A is $m \times n$, AB defined therefore B should be $n \times$ p because the number of columns of A should be equal to number of rows of B.

Again B is now $n \times$ p and A is $m \times n$.

And since BA is also defined therefore p would be equal to m by the same argument as above.

$\therefore$ B is $n \times m$ matrix.

Illustration 6

Question: **If α and β differ by an odd multiple of $\pi/2$ prove that the product of the two matrices given below is a null matrix.**

$$A = \begin{bmatrix} \cos^2\alpha & \cos\alpha\sin\alpha \\ \cos\alpha\sin\alpha & \sin^2\alpha \end{bmatrix}$$

$$B = \begin{bmatrix} \cos^2\beta & \cos\beta\sin\beta \\ \cos\beta\sin\beta & \sin^2\beta \end{bmatrix}$$

Solution:

$$AB = \begin{bmatrix} \cos\alpha\cos\beta\cos(\alpha-\beta) & \cos\alpha\sin\beta\cos(\alpha-\beta) \\ \sin\alpha\cos\beta\cos(\alpha-\beta) & \sin\alpha\sin\beta\cos(\alpha-\beta) \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$\therefore$ $\alpha - \beta$ = odd multiple of $\pi/2$

$\therefore$ $\cos(\alpha - \beta) = 0$.

7.4 PROPERTIES OF MATRIX MULTIPLICATION

(a) Multiplication of matrices is distributive with respect to addition of matrices

i.e. $A(B + C) = AB + AC$.

(b) Matrix multiplication is associative if conformability is assured.

i.e. $A(BC) = (AB)C$.

(c) The multiplication of matrices is not always commutative.

i.e. AB is not always equal to BA.

(d) Multiplication of a matrix A by a null matrix conformable with A, will give null-matrix.

i.e. Let $A = \begin{bmatrix} 2 & 3 & 4 \\ 5 & 1 & 2 \\ 6 & 4 & 2 \\ 7 & 4 & 6 \end{bmatrix}_{4\times 3}$ and $O = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}_{3\times 2}$ $\Rightarrow$ $AO = O$

8. OPERATIONS REGARDING MATRICES

8.1 TRANSPOSE OF MATRIX

If A be a given matrix of the type $m \times n$ then the matrix obtained by changing the rows of A into columns and columns of A into rows is called transpose of matrix A and is denoted by A' or A^T. As there are m rows in A therefore there will be m columns in A' and similarly as there are n columns in A there will be n rows in A'.

Properties of transpose

(i) $(A')' = A$

(ii) $(KA)' = KA'$. K being a scalar.

(iii) $(A \pm B)' = A' \pm B'$

(iv) $(AB)' = B' A'$.

(v) $(ABC)' = C' B' A'$.

***Illustration* 7**

Question: **Taking any matrix of order 3×3 prove that $(A')' = A$.**

Solution: Let a matrix $A = \begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix}$

y $A' = \begin{bmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{bmatrix}$

Again $(A')' = \begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix} = A$

$\therefore \quad (A')' = A$

9. TYPES OF MATRIX ON THE BASIS OF OPERATIONS

9.1 SYMMETRIC MATRIX

A square matrix $A = [a_{ij}]$ is said to be symmetric, if its $(i, j)^{th}$ element is the same as its (j, i)th element i.e., if $a_{ij} = a_{ji}$ for all i, j.

9.2 SKEW SYMMETRIC MATRIX

A square matrix $A = [a_{ij}]$ is said to be skew symmetric if the $(i, j)^{th}$ element of A is the negative of the (j, i)th element of A i.e., if $a_{ij} = -a_{ji}$ for all i, j.

Illustration 8

Question: **Show that the elements on the main diagonal of a skew-symmetric matrix are all zero.**

Solution: Let $A = [a_{ij}]$ be a skew-symmetric matrix. Then

$a_{ij} = -a_{ji}$ for all i, j

$\Rightarrow \quad a_{ii} = -a_{ii}$ for all values of i

$\Rightarrow \quad 2a_{ii} = 0 \Rightarrow a_{ii} = 0$ for all values of i

$\Rightarrow \quad a_{11} = a_{22} = a_{33} = ... = a_{nn} = 0$

10. ELEMENTARY OPERATION (Transformation) OF A MATRIX

There are six operations (transformation) on a matrix, three of which are due to rows and three due to columns, which are known as elementary operations or transformations.

(i) The interchange of any two rows or two columns. Symbolically the interchange of i^{th} and jth rows is denoted by $R_i \leftrightarrow R_j$ and interchange of i^{th} and jth column is denoted by $C_i \leftrightarrow C_j$.

(ii) The multiplication of the elements of any row or column by a non zero number. Symbolically, the multiplication of each element of the ith row by k, where $k \neq 0$ is denoted by $R_i \to kR_j$.

The corresponding column operation is denoted by $C_i \to kC_i$

(iii) The addition to the elements of any row or column, the corresponding elements of any other row or column multiplied by any non-zero number. Symbolically, the addition to the elements of ith row, the corresponding elements of jth row multiplied by k is denoted by $R_i \to R_i + kR_j$.

The corresponding column operation is denoted by $C_i \to C_i + kC_j$

11. INVERTIBLE MATRICES

If *A* is a square matrix of order *m* and if there exists another square matrix *B* of the same order *m*, such that *AB* = *BA* = *I*, then *B* is called the inverse matrix of *A* and it is denoted by A^{-1}. In that case *A* is said to be invertible.

Notes: (i) A rectangular matrix does not posses inverse matrix, since for products *BA* and *AB* to be defined and to be equal, it is necessary that matrices *A* and *B* should be square matrices of the same order.

(ii) If *B* is the inverse of *A*, then *A* is also the inverse of *B*.

12. INVERSE OF A MATRIX BY ELEMETNARY OPERATIONS

Let *X*, *A* and *B* be matrices of the same order such that $X = AB$. In order to apply a sequence of elementary row operation on the matrix equation $X = AB$, we will apply these row operations simultaneously on *X* and on the first matrix *A* of the product *AB* on RHS.

Similarly, in order to apply a sequence of elementary column operations on the matrix equation $X = AB$, we will apply these operations simultaneously on *X* and on the second matrix *B* of the product *AB* on RHS.

IN view of the above discussion, we conclude that if *A* is a matrix such that A^{-1} exists, then to find A^{-1} using elementary row operations, write $A = IA$ and apply a sequence of row operation on $A = IA$ till we get $I = BA$. The matrix *B* will be the inverse of *A*. Similarly, if we wish to find A^{-1} using column operations, then write $A = AI$ and apply a sequence of column operations on $A = AI$ till we get $I = AB$.

Remark: In case, after applying one or more elementary row (column) operations on $A = IA(A = AI)$, if we obtain all zeros in one or more rows of the matrix *A* or LHS, then A^{-1} does not exist.

***Illustration* 9**

Question: **By using elementary operations, find the inverse of the matrix** $A = \begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix}$.

Solution: In order to use elementary row operations we may write $A = IA$.

or $\begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$, then $\begin{bmatrix} 1 & 2 \\ 0 & -5 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix} A$ (applying $R_2 \to R_2 - 2R_1$)

or $\begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 2/5 & -1/5 \end{bmatrix} A$ (applying $R_2 \to -\frac{1}{5} R_2$)

or $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1/5 & 2/5 \\ 2/5 & -1/5 \end{bmatrix} A$ (applying $R_1 \to R_1 - 2R_2$)

Thus $A^{-1} = \begin{bmatrix} 1/5 & 2/5 \\ 2/5 & -1/5 \end{bmatrix}$

PRACTICE PROBLEMS

PP1. Using elementary transformation, find the inverse of $\begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}$.

PP2. If $\begin{bmatrix} x+3 & 2y+x \\ z-1 & 4a-6 \end{bmatrix} = \begin{bmatrix} 0 & -7 \\ 3 & 2a \end{bmatrix}$ then find value of *x, y, z*.

PP3. If $A = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & -1 \\ 3 & -1 & 1 \end{bmatrix}$, then find value of $A^3 - 3A^2 - A + 9I$.

PP4. If $3A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ x & 2 & y \end{bmatrix}$ and $A.A' = I$, then find $x + y$.

PP5. For the three matrices *A*, *B*, *C*, $A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, B = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}, C = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$ verify the following relations:
$A^2 = B^2 = C^2 = I$
$AB = -BA$; $AC = -CA$; $BC = -CB$.

PP6. If $A = \begin{bmatrix} 1 & 0 \\ -1 & 7 \end{bmatrix}$ and $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, then find *k* so that $A^2 = 8A + kI$.

PP7. If $A = \begin{bmatrix} 0 & c & -b \\ -c & 0 & a \\ b & -a & 0 \end{bmatrix}$ and $B = \begin{bmatrix} a^2 & ab & ac \\ ab & b^2 & bc \\ ac & bc & c^2 \end{bmatrix}$, show that $AB = BA = O_{3\times3}$.

PP8. Show that the matrix $A = \begin{bmatrix} 5 & 3 \\ 12 & 7 \end{bmatrix}$ is a root of the equation $A^2 - 12A - I = 0$

PP9. Let *A* and *B* be symmetric matrices of the same order. Then, show that
(i) $AB - BA$ is a skew symmetric matrix
(ii) $AB + BA$ is a symmetric matrix

PP10. If $A = \begin{bmatrix} 3 & -4 \\ 1 & 1 \end{bmatrix}, B = \begin{bmatrix} 2 & 1 & 2 \\ 1 & 3 & 4 \end{bmatrix}$, show that $(AB)^T = B^T A^T$.

SOLVED SUBJECTIVE EXAMPLES

***Example* 1:**

For what values of *x* and *y* are the following matrices equal

$$A = \begin{bmatrix} 2x+1 & 3y \\ 0 & y^2-5y \end{bmatrix},\quad B = \begin{bmatrix} x+3 & y^2+2 \\ 0 & -6 \end{bmatrix}.$$

Solution:

Since the corresponding elements of two matrices are equal, therefore,

$A = B \Rightarrow 2x+1 = x+3,\ 3y = y^2+2$ and $y^2-5y = -6$

Now, $2x+1 = x+3 \Rightarrow x = 2$, $3y = y^2+2$

$\Rightarrow y^2-3y+2 = 0 \Rightarrow y = 1, 2$

and $y^2-5y = -6 \Rightarrow y^2-5y+6 = 0 \Rightarrow y = 2, 3$

Since $3y = y^2+2$ and $y^2-5y = -6$ must hold good simultaneously, we take the common solution of these two equations. Therefore $y = 2$.

Hence $A = B$ if $x = 2,\ y = 2$

***Example* 2:**

Simplify $\cos\theta\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} + \sin\theta\begin{bmatrix} \sin\theta & -\cos\theta \\ \cos\theta & \sin\theta \end{bmatrix}$.

Solution:

We have, $\cos\theta\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} + \sin\theta\begin{bmatrix} \sin\theta & -\cos\theta \\ \cos\theta & \sin\theta \end{bmatrix}$

$$= \begin{bmatrix} \cos^2\theta & \sin\theta\cos\theta \\ -\sin\theta\cos\theta & \cos^2\theta \end{bmatrix} + \begin{bmatrix} \sin^2\theta & -\sin\theta\cos\theta \\ \sin\theta\cos\theta & \sin^2\theta \end{bmatrix}$$

$$= \begin{bmatrix} \cos^2\theta+\sin^2\theta & \sin\theta\cos\theta-\sin\theta\cos\theta \\ -\sin\theta\cos\theta+\sin\theta\cos\theta & \cos^2\theta+\sin^2\theta \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

***Example* 3:**

Let $A = \begin{bmatrix} 1 & -2 & 3 \\ 3 & 2 & -1 \end{bmatrix}$ **and** $B = \begin{bmatrix} 2 & 3 \\ -1 & 2 \\ 4 & -5 \end{bmatrix}$. **Find *AB* and *BA* and show that** $AB \neq BA$.

Solution:

We have $AB = \begin{bmatrix} 1 & -2 & 3 \\ 3 & 2 & -1 \end{bmatrix}\begin{bmatrix} 2 & 3 \\ -1 & 2 \\ 4 & -5 \end{bmatrix} \Rightarrow AB = \begin{bmatrix} 2+2+12 & 3-4-15 \\ 6-2-4 & 9+4+5 \end{bmatrix} = \begin{bmatrix} 16 & -16 \\ 0 & 18 \end{bmatrix}$

Again, *B* is a 3×2 matrix and *A* is a 2×3 matrix.

So, *BA* exists and it is or order 3×3.

Now, $BA = \begin{bmatrix} 2 & 3 \\ -1 & 2 \\ 4 & -5 \end{bmatrix} \begin{bmatrix} 1 & -2 & 3 \\ 3 & 2 & -1 \end{bmatrix}$

$\Rightarrow \quad BA = \begin{bmatrix} 2+9 & -4+6 & 6-3 \\ -1+6 & 2+4 & -3-2 \\ 4-15 & -8-10 & 12+5 \end{bmatrix} = \begin{bmatrix} 11 & 2 & 3 \\ 5 & 6 & -5 \\ -11 & -18 & 17 \end{bmatrix}$

Clearly, $AB \neq BA$

Example 4:

If $A = \begin{bmatrix} \alpha & 0 \\ 1 & 1 \end{bmatrix}$ **and** $B = \begin{bmatrix} 1 & 0 \\ 5 & 1 \end{bmatrix}$**, find the values of ~ for which A^2 Z B.**

Solution:

We have $A^2 = B$

$\Rightarrow \quad \begin{bmatrix} \alpha & 0 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} \alpha & 0 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 5 & 1 \end{bmatrix}$

$\Rightarrow \quad \begin{bmatrix} \alpha^2+1 & 0+0 \\ \alpha+1 & 0+1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 5 & 1 \end{bmatrix} \Rightarrow \begin{bmatrix} \alpha^2 & 0 \\ \alpha+1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 5 & 1 \end{bmatrix}$

$\Rightarrow \quad \alpha^2 = 1$ and $\alpha + 1 = 5$

$\Rightarrow \quad \alpha = \pm 1$ and $\alpha = 4$, which is not possible.

Example 5:

If $\begin{bmatrix} 2 & -1 \\ 1 & 0 \\ -3 & 4 \end{bmatrix} A = \begin{bmatrix} -1 & -8 & -10 \\ 1 & -2 & -5 \\ 9 & 22 & 15 \end{bmatrix}$**, find *A*.**

Solution:

Since the product matrix is a 3×3 matrix and the per multiplier of A is a 3×2 matrix.

Therefore A is 2×3 matrix.

Let $A = \begin{bmatrix} x & y & z \\ a & b & c \end{bmatrix}$. Then the given equation becomes

$\begin{bmatrix} 2 & -1 \\ 1 & 0 \\ -3 & 4 \end{bmatrix} \begin{bmatrix} x & y & z \\ a & b & c \end{bmatrix} = \begin{bmatrix} -1 & -8 & -10 \\ 1 & -2 & -5 \\ 9 & 22 & 15 \end{bmatrix}$

$\Rightarrow \quad \begin{bmatrix} 2x-a & 2y-b & 2z-c \\ x & y & z \\ -3x+4a & -3y+4b & -3z+4c \end{bmatrix} = \begin{bmatrix} -1 & -8 & -10 \\ 1 & -2 & -5 \\ 9 & 22 & 15 \end{bmatrix}$

$\Rightarrow \quad 2x - a = -1,\ x = 1,\ -3x + 4a = 9,\ 2y - b = -8,\ y = -2,$

$-3y + 4b = 22,\ 2z - c = -10,\ z = -5,\ -3z + 4c = 15$

$\Rightarrow \quad x = 1,\ a = 3,\ y = -2,\ b = 4,\ z = -5$ and $c = 0$

Hence $A=\begin{bmatrix}1 & -2 & -5\\ 3 & 4 & 0\end{bmatrix}$

***Example* 6:**

Let $f(x)=x^2-5x+6$. Find $f(A)$ if $A=\begin{bmatrix}2 & 0 & 1\\ 2 & 1 & 3\\ 1 & -1 & 0\end{bmatrix}$.

Solution:

First, we note that by $f(A)$ we mean the matrix polynomial $A^2-5A+6I_3$.
That is, to obtain $f(A)$, x is replaced by A and the constant term is multiplied by the indentity matrix of order same as that of A.

Now, $A^2=AA=\begin{bmatrix}2 & 0 & 1\\ 2 & 1 & 3\\ 1 & -1 & 0\end{bmatrix}\begin{bmatrix}2 & 0 & 1\\ 2 & 1 & 3\\ 1 & -1 & 0\end{bmatrix}$

$\Rightarrow \quad A^2=\begin{bmatrix}4+0+1 & 0+0-1 & 2+0+0\\ 4+2+3 & 0+1-3 & 2+3+0\\ 2-2+0 & 0-1+0 & 1-3+0\end{bmatrix}=\begin{bmatrix}5 & -1 & 2\\ 9 & -2 & 5\\ 0 & -1 & -2\end{bmatrix}$

$\Rightarrow \quad -5A=\begin{bmatrix}-10 & .0 & -5\\ -10 & -5 & -15\\ -5 & 5 & 0\end{bmatrix}$

$\Rightarrow \quad 6I_3=6\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}=\begin{bmatrix}6 & 0 & 0\\ 0 & 6 & 0\\ 0 & 0 & 6\end{bmatrix}$

$\therefore \quad f(A)=A^2-5A+6I_3=\begin{bmatrix}5 & -1 & 2\\ 9 & -2 & 5\\ 0 & -1 & -2\end{bmatrix}+\begin{bmatrix}-10 & 0 & -5\\ -10 & -5 & -15\\ -5 & 5 & 0\end{bmatrix}+\begin{bmatrix}6 & 0 & 0\\ 0 & 6 & 0\\ 0 & 0 & 6\end{bmatrix}$

$\Rightarrow \quad f(A)=A^2-5A+6I_3=\begin{bmatrix}1 & -1 & -3\\ -1 & -1 & -10\\ -5 & 4 & 4\end{bmatrix}$.

***Example* 7:**

Use matrix multiplication to divide Rs. 30,000 in two parts such that the total annual interest at 9% on the first part and 11% on the second part amounts Rs. 3060.

Solution:

Let the two parts be Rs x and Rs. (30000 – x) respectively.
Let A be the 1 × 2 matrix representing these two parts

i.e., $\quad \begin{matrix} & \text{Part-I} & \text{Part-II}\\ A= & [x & 30000-x]\end{matrix}$

Let R denote the 2 × 1 matrix representing the annual interest rates of interest on two parts i.e.,

$R=\begin{matrix}\text{Part}-\text{I}\\ \text{Part}-\text{II}\end{matrix}\begin{bmatrix}0.09\\ 0.11\end{bmatrix}$

The total annual interest on the two parts is given by the matrix multiplication AR.

$\therefore \quad AR = 3060$

$\Rightarrow \quad [x \quad 30000 - x]\begin{bmatrix} 0.09 \\ 0.11 \end{bmatrix} = 3060$

$\Rightarrow \quad [0.09x + 0.11(30000 - x) = 3060]$

$\Rightarrow \quad \dfrac{9}{100}x + \dfrac{11}{100}(30000 - x) = 3060$

$\Rightarrow \quad 9x + 330000 - 11x = 306000 \quad \Rightarrow \quad x = 12{,}000$

Hence two parts of Rs. 30,000 are Rs. 12,000 and Rs. 18,000 respectively.

Example 8:

If $A = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$, then find the values of θ satisfying the equation $A^T + A = I_2$.

Solution:

We have, $A = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$

$\therefore \quad A^T = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$

Now, $A^T + A = I_2$

$\Rightarrow \quad \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} + \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \Rightarrow \begin{bmatrix} 2\cos\theta & 0 \\ 0 & 2\cos\theta \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\Rightarrow \quad 2\cos\theta = 1 \quad \Rightarrow \quad \cos\theta = \dfrac{1}{2} \quad \Rightarrow \quad \cos\theta = \cos\dfrac{\pi}{3} \quad \Rightarrow \quad \theta = 2n\pi \pm \dfrac{\pi}{3},\ n \in Z$

Example 9:

Find the values of *x, y, z* if the matrix $A = \begin{bmatrix} 0 & 2y & z \\ x & y & -z \\ x & -y & z \end{bmatrix}$ satisfy the equation $A^T A = I_3$.

Solution:

We have, $A = \begin{bmatrix} 0 & 2y & z \\ x & y & -z \\ x & -y & z \end{bmatrix} \Rightarrow A^T = \begin{bmatrix} 0 & x & x \\ 2y & y & -y \\ z & -z & z \end{bmatrix}$

$\therefore \quad A^T A = I_3$

$\Rightarrow \quad \begin{bmatrix} 0 & x & x \\ 2y & y & -y \\ z & -z & z \end{bmatrix}\begin{bmatrix} 0 & 2y & z \\ x & y & -z \\ x & -y & z \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \Rightarrow \begin{bmatrix} 2x^2 & 0 & 0 \\ 0 & 6y^2 & 0 \\ 0 & 0 & 3z^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$\Rightarrow \quad 2x^2 = 1,\ 6y^2 = 1,\ 3z^2 = 1 \Rightarrow \quad x = \pm\dfrac{1}{\sqrt{2}},\ y = \pm\dfrac{1}{\sqrt{6}},\ z = \pm\dfrac{1}{\sqrt{3}}$

***Example* 10:**

Prove that every square matrix can be uniquely expressed as the sum of a symmetric matrix and a skew-symmetric matrix.

Solution:

Let A be a square matrix. Then $A = \frac{1}{2}\left(A + A^T\right) + \frac{1}{2}\left(A - A^T\right) = P + Q$

where $P = \frac{1}{2}\left(A + A^T\right)$ and $Q = \frac{1}{2}\left(A - A^T\right)$

Now $P^T = \left(\frac{1}{2}\left(A + A^T\right)\right)^T = \frac{1}{2}\left(A + A^T\right)^T$ [$\because$ $(kA)^T = kA^T$]

$\Rightarrow$ $P^T = \frac{1}{2}\left(A^T + \left(A^T\right)^T\right)$ [$\because$ $(A+B)^T = A^T + B^T$]

$\Rightarrow$ $P^T = \frac{1}{2}\left(A^T + A\right)$ [$\because$ $\left(A^T\right)^T = A$]

$\Rightarrow$ $P^T = \frac{1}{2}\left(A + A^T\right) = P$

$\therefore$ P is a symmetric matrix.

Also, $Q^T = \left(\frac{1}{2}\left(A - A^T\right)\right)^T = \frac{1}{2}\left(A - A^T\right)^T = \frac{1}{2}\left(A^T - \left(A^T\right)^T\right)$

$\Rightarrow$ $Q^T = \frac{1}{2}\left(A^T - A\right) = -\frac{1}{2}\left(A - A^T\right) = -Q$

$\therefore$ Q is a skew-symmetric matrix.

Thus, $A = P + Q$, where P is a symmetric matrix and Q is a skew-symmetric matrix.

Hence A is expressible as the sum of a symmetric and a skew-symmetric matrix.

EXERCISE – I

1. If $A=\begin{bmatrix}2 & 3\\ 1 & 2\\ 5 & 6\end{bmatrix}, B=\begin{bmatrix}3 & 2\\ 5 & 6\\ 2 & 1\end{bmatrix}$ and $A + B - C = 0$, then find C.

2. If $\begin{bmatrix}5 & k+2\\ k+1 & -2\end{bmatrix}=\begin{bmatrix}k+3 & 4\\ 3 & -k\end{bmatrix}$ then find the value of k.

3. If $A=\begin{bmatrix}1 & 0\\ 2 & 0\end{bmatrix}, B=\begin{bmatrix}0 & 0\\ 1 & 12\end{bmatrix}$, then prove that $AB = O$, $BA \neq O$.

4. If $A=\begin{bmatrix}1 & 0 & 1\\ 0 & 0 & 0\\ 1 & 0 & 1\end{bmatrix}$, then find A^2.

5. If $A=\begin{bmatrix}4 & 2\\ -1 & 1\end{bmatrix}$, then find $(A - 2I)(A - 3I)$.

6. If $A=\begin{bmatrix}1 & 0\\ 2 & -1\end{bmatrix}, B=\begin{bmatrix}3 & 7\\ 4 & 8\end{bmatrix}, C=\begin{bmatrix}-1 & 1\\ 0 & 0\end{bmatrix}$ verify that $A + (B + C) = (A + B) + C$.

7. If $A=\begin{bmatrix}5 & 0\\ 0 & 5\end{bmatrix}$ and $B=\begin{bmatrix}a & b\\ c & d\end{bmatrix}$, then find AB.

8. If $A=\begin{bmatrix}1 & 3\\ 3 & 4\end{bmatrix}$ and $A^2 - kA - 5I_2 = 0$, then find the value of k.

9. Find the value of x and y from the equation $2\begin{bmatrix}x & 5\\ 7 & y-3\end{bmatrix}+\begin{bmatrix}3 & 4\\ 1 & 2\end{bmatrix}=\begin{bmatrix}7 & 14\\ 15 & 14\end{bmatrix}$.

10. If $A=\begin{bmatrix}3 & -2\\ 4 & -2\end{bmatrix}$ and $I=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$, find k, such that $A^2 = kA - 2I$.

11. If $A=\begin{bmatrix}3 & -4\\ 7 & 8\end{bmatrix}$, show that $A - A'$ is a skew-symmetric matrix and $A + A'$ is symmetric matrix.

12. If A is 3 × 3 matrix and B is a matrix such that $A'B$ and BA' are both defined. Then find order of B.

13. If $A=\begin{bmatrix}1 & 2\\0 & 1\end{bmatrix}$, then find A^n.

14. If A and B are square matrices of size $n \times n$ such that $A^2 - B^2 = (A-B)(A+B)$, then prove that $AB = BA$.

15. Find the value of x for which the matrix product $\begin{bmatrix}2 & 1 & 7\\0 & 0 & 0\\1 & -2 & 1\end{bmatrix}\begin{bmatrix}-x & 14x & 7x\\0 & 1 & 0\\x & -4x & -2x\end{bmatrix}$ equal an identity matrix.

EXERCISE – II

1. Find the value of $\begin{bmatrix} 2 & 3 & 4 \\ 3 & 4 & 5 \\ 4 & 5 & 6 \end{bmatrix}\begin{bmatrix} 1 & -3 & 5 \\ 0 & 2 & 4 \\ 3 & 0 & 5 \end{bmatrix}$.

2. If U and V are two symmetric matrices, show that UVU is also symmetric. Is UV symmetric always? Explain and illustrate by an example.

3. If $A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$, show that $A^2 - 4A - 5I = O$.

4. If $A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$, prove that $(aI + bA)^n = a^n I + na^{n-1} bA$, where I is the two rowed unit matrix and n is a positive integer.

5. Let $f(x) = x^2 - 5x + 6$, find $f(A)$ i.e., $A^2 = 5A + 6I$, if $A = \begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix}$.

6. If $A = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$, prove that $A^n = \begin{bmatrix} 1+2n & -4n \\ n & 1-2n \end{bmatrix}$, $n \in N$.

7. Using elementary transformation find the inverse of $\begin{bmatrix} 2 & -3 & 3 \\ 2 & 2 & 3 \\ 3 & -2 & 2 \end{bmatrix}$.

8. Find the value of x, such that $[1\ 2\ 1]\begin{bmatrix} 1 & 2 & 0 \\ 2 & 0 & 1 \\ 1 & 0 & 2 \end{bmatrix}\begin{bmatrix} 0 \\ 2 \\ x \end{bmatrix} = O$.

9. If $x = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$, then find x^n, $n \in N$.

10. Find the inverse of the matrices $\begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$ using elementary row method.

11. If $[1\ \ x\ \ 1]\begin{bmatrix} 1 & 3 & 2 \\ 0 & 5 & 1 \\ 0 & 3 & 2 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \\ x \end{bmatrix} = O$, then find x.

12. A trust fund has Rs. 30000 that must be invested in two different types of bonds. The first bond pays 5% interest per year and the second bond pays 7% interest per year. Using matrix multiplication, determine how to divide Rs 30000 among the two types of bonds. If the trust fund must obtain an annual total interest of

(i) Rs. 1800 (ii) Rs. 2000

13. If $A=[x\ y\ z]$, $B=\begin{bmatrix} a & h & g \\ h & b & f \\ g & f & c \end{bmatrix}$ and $C=[x\ y\ z]^t$, then find the order of ABC.

14. If $A=\begin{bmatrix} 0 & 5 \\ 0 & 0 \end{bmatrix}$ and $f(x)=I+x+x^2+......+x^{16}$, then find $f(A)$.

15. Let $A=\begin{bmatrix} 2 & -1 \\ 3 & 4 \end{bmatrix}$, $B=\begin{bmatrix} 5 & 2 \\ 7 & 4 \end{bmatrix}$, $C=\begin{bmatrix} 2 & 5 \\ 3 & 8 \end{bmatrix}$. Find a matrix D such that $CD-AB=O$.

ANSWERS

ANSWERS TO PRACTICE PROBLEMS

PP1. $\begin{bmatrix} 3/5 & 1/5 \\ -2/5 & 1/5 \end{bmatrix}$

PP2. $x = -3$, $y = -2$, and $z = 4$

PP3. zero

PP4. -3

PP6. $k = -7$

PP8. $A = \begin{bmatrix} 3 & 3 & 5/2 \\ 3 & 5 & 7/2 \\ 5/2 & 7/2 & 5 \end{bmatrix} + \begin{bmatrix} 0 & -1 & 1/2 \\ 1 & 0 & -1/2 \\ -1/2 & 1/2 & 0 \end{bmatrix}$

EXERCISE – I

1. $\begin{bmatrix} 5 & 5 \\ 6 & 8 \\ 7 & 7 \end{bmatrix}$

2. 2

4. $2A$

5. O

7. $5B$

8. 5

9. $x = 2,\ y = 9$

10. $k = 1$

12. 3×3

13. $\begin{bmatrix} 1 & 2n \\ 0 & 1 \end{bmatrix}$

15. $\frac{1}{5}$

EXERCISE – II

1. $\begin{bmatrix} 14 & 0 & 42 \\ 18 & -1 & 56 \\ 22 & -2 & 70 \end{bmatrix}$

5. $\begin{bmatrix} 1 & -1 & -3 \\ -1 & -1 & -10 \\ -5 & 4 & 4 \end{bmatrix}$

7. $\begin{bmatrix} -2/5 & 0 & 3/5 \\ -1/5 & 1/5 & 0 \\ 2/5 & 1/5 & -2/5 \end{bmatrix}$

8. $x = -1$

9. $2^{n-1}x$

10. $\begin{bmatrix} 1/2 & -1/2 & 1/2 \\ -4 & 3 & -1 \\ 5/2 & -3/2 & 1/2 \end{bmatrix}$

11. $\dfrac{-9 \pm \sqrt{53}}{2}$

12. (i) Rs. 15000 each (ii) Rs. 5000, Rs. 25000

13. 1 × 1

14. $\begin{bmatrix} 1 & 5 \\ 0 & 1 \end{bmatrix}$

15. $D = \begin{bmatrix} -191 & -110 \\ 77 & 44 \end{bmatrix}$