



# Wave Motion

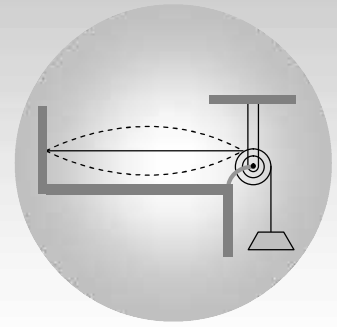
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# Wave Motion

## 16.1 Wave

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A wave is a disturbance which propagates energy and momentum from one place to the other without the transport of matter.

(1) **Necessary properties of the medium for wave propagation :**

(i) Elasticity : So that particles can return to their mean position, after having been disturbed.

(ii) Inertia : So that particles can store energy and overshoot their mean position.

(iii) Minimum friction amongst the particles of the medium.

(iv) Uniform density of the medium.

(2) **Characteristics of wave motion :**

(i) It is a sort of disturbance which travels through a medium.

(ii) Material medium is essential for the propagation of mechanical waves.

(iii) When a wave motion passes through a medium, particles of the medium only vibrate simple harmonically about their mean position. They do not leave their position and move with the disturbance.

(iv) There is a continuous phase difference amongst successive particles of the medium *i.e.*, particle 2 starts vibrating slightly later than particle 1 and so on.

(v) The velocity of the particle during their vibration is different at different positions.

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(vi) The velocity of wave motion through a particular medium is constant. It depends only on the nature of medium not on the frequency, wavelength or intensity.

(vii) Energy is propagated along with the wave motion without any net transport of the medium.

(3) **Mechanical waves** : The waves which require medium for their propagation are called mechanical waves.

*Example* : Waves on string and spring, waves on water surface, sound waves, seismic waves.

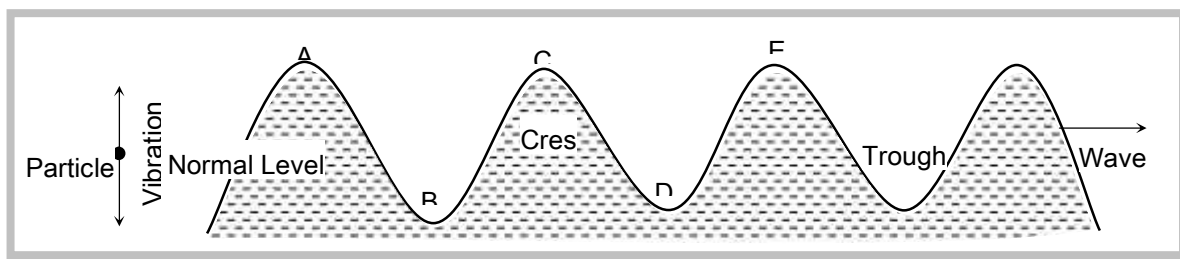
(4) **Non-mechanical waves** : The waves which do not require medium for their propagation are called non-mechanical or electromagnetic waves.

*Examples* : Light, heat (Infrared), radio waves, x-rays, X-rays etc.

(5) **Transverse waves** : Particles of the medium execute simple harmonic motion about their mean position in a direction perpendicular to the direction of propagation of wave motion.

(i) It travels in the form of crests and troughs.

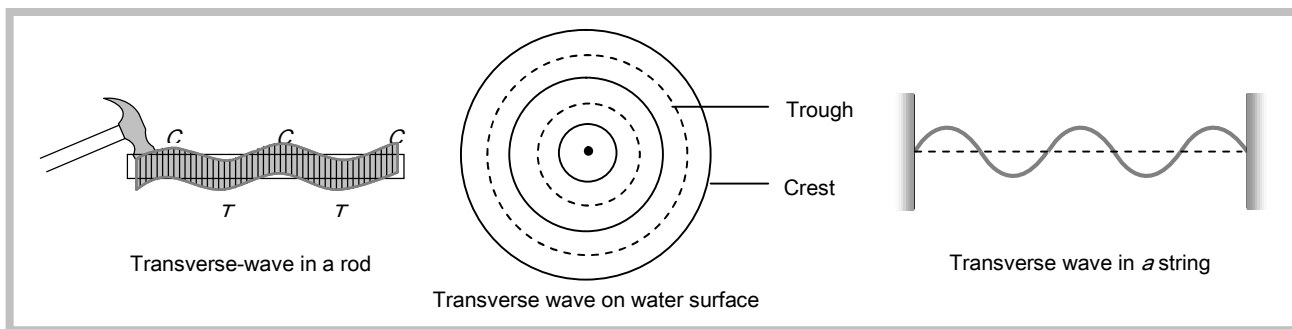
(ii) A crest is a portion of the medium which is raised temporarily above the normal position of rest of the particles of the medium when a transverse wave passes through it.



(iii) A trough is a portion of the medium which is depressed temporarily below the normal position of rest of the particles of the medium, when transverse wave passes through it.

(iv) Examples of transverse wave motion : Movement of string of a sitar or violin, movement of the membrane of a Tabla or Dholak, movement of kink on a rope, waves set-up on the surface of water.

(v) Transverse waves can be transmitted through solids, they can be setup on the surface of liquids. But they can not be transmitted into liquids and gases.



(6) **Longitudinal waves** : If the particles of a medium vibrate in the direction of wave motion the wave is called longitudinal.

(i) It travels in the form of compression and rarefaction.

(ii) A compression ( $C$ ) is a region of the medium in which particles are compressed.

(iii) A rarefaction ( $R$ ) is a region of the medium in which particles are rarefied.

(iv) Examples sound waves travel through air in the form of longitudinal waves, Vibration of air column in organ pipes are longitudinal, Vibration of air column above the surface of water in the tube of resonance apparatus are longitudinal.

(v) These waves can be transmitted through solids, liquids and gases because for these waves propagation, volume elasticity is necessary.

(7) **One dimensional wave** : Energy is transferred in a single direction only.

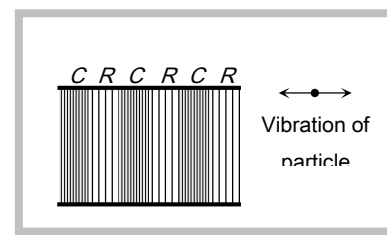
*Example* : Wave propagating in a stretched string.

(8) **Two dimensional wave** : Energy is transferred in a plane in two mutually perpendicular directions.

*Example* : Wave propagating on the surface of water.

(9) **Three dimensional wave** : Energy is transferred in space in all direction.

*Example* : Light and sound waves propagating in space.



## 16.2 Important Terms Regarding Wave Motion

(1) **Wavelength** : (i) It is the length of one wave.

(ii) Wavelength is equal to the distance travelled by the wave during the time in which any one particle of the medium completes one vibration about its mean position.

(iii) Wavelength is the distance between any two nearest particles of the medium, vibrating in the same phase.

(iv) Distance travelled by the wave in one time period is known as wavelength.

(v) In transverse wave motion :

} = Distance between the centres of two consecutive crests.

} = Distance between the centres of two consecutive troughs.

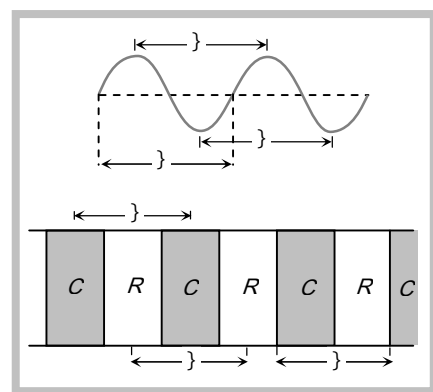
} = Distance in which one trough and one crest are contained.

(vi) In longitudinal wave motion :

} = Distance between the centres of two consecutive compression.

} = Distance between the centres of two consecutive rarefaction.

} = Distance in which one compression and one rarefaction contained.



(2) **Frequency** : (i) Frequency of vibration of a particle is defined as the number of vibrations completed by particle in one second.

(ii) It is the number of complete wavelengths traversed by the wave in one second.

(iii) Units of frequency are hertz ( $Hz$ ) and per second.

(3) **Time period** : (i) Time period of vibration of particle is defined as the time taken by the particle to complete one vibration about its mean position.

(ii) It is the time taken by the wave to travel a distance equal to one wavelength.

(4) **Relation between frequency and time period** : Time period =  $1/\text{Frequency} \Rightarrow T = 1/n$

(5) **Relation between velocity, frequency and wavelength** :  $v = n\lambda$

Velocity ( $v$ ) of the wave in a given medium depends on the elastic and inertial property of the medium. Frequency ( $n$ ) is characterised by the source which produces disturbance. Different sources may produce vibration of different frequencies. Wavelength ( $\lambda$ ) will differ to keep  $n\lambda = v = \text{constant}$

### 16.3 Sound Waves

The energy to which the human ears are sensitive is known as sound. In general all types of waves are produced in an elastic material medium, Irrespective of whether these are heard or not are known as sound.

According to their frequencies, waves are divided into three categories :

(1) **Audible or sound waves** : Range 20 Hz to 20 KHz. These are generated by vibrating bodies such as vocal cords, stretched strings or membrane.

(2) **Infrasonic waves** : Frequency lie below 20 Hz.

*Example* : waves produced during earth quake, ocean waves *etc.*

(3) **Ultrasonic waves** : Frequency greater than 20 KHz. Human ear cannot detect these waves, certain creatures such as mosquito, dog and bat show response to these. As velocity of sound in air is 332 m/sec so the wavelength of ultrasonics  $\lambda < 1.66 \text{ cm}$  and for infrasonics  $\lambda > 16.6 \text{ m}$ .

**Note** :  **Supersonic speed** : An object moving with a speed greater than the speed of sound is said to move with a supersonic speed.

**Mach number** : It is the ratio of velocity of source to the velocity of sound.

$$\text{Mach Number} = \frac{\text{Velocity of source}}{\text{Velocity of sound}}$$

**Difference between sound and light waves** :

(i) For propagation of sound wave material medium is required but no material medium is required for light waves.

(ii) Sound waves are longitudinal but light waves are transverse.

(iii) Wavelength of sound waves ranges from 1.65 cm to 16.5 meter and for light it ranges from 4000 Å to 2000 Å.

### 16.4 Velocity of Sound (Wave motion)

(1) Speed of transverse wave motion :

(i) On a stretched string :  $v = \sqrt{\frac{T}{m}}$   $T$  = Tension in the string;  $m$  = Linear density of string (mass per unit length).

(ii) In a solid body :  $v = \sqrt{\frac{y}{\dots}}$   $y$  = Modulus of rigidity;  $\dots$  = Density of the material.

(2) Speed of longitudinal wave motion:

(i) In a solid medium  $v = \sqrt{\frac{k + \frac{4}{3}y}{\dots}}$   $k$  = Bulk modulus;  $y$  = Modulus of rigidity;  $\dots$  = Density

When the solid is in the form of long bar  $v = \sqrt{\frac{Y}{\dots}}$   $Y$  = Young's modulus of material of rod

(ii) In a liquid medium  $v = \sqrt{\frac{k}{\dots}}$

(iii) In gases  $v = \sqrt{\frac{k}{\dots}}$

### 16.5 Velocity of Sound in Elastic Medium

When a sound wave travels through a medium such as air, water or steel, it will set particles of medium into vibration as it passes through it. For this to happen the medium must possess both inertia *i.e.* mass density (so that kinetic energy may be stored) and elasticity (so that PE may be stored). These two properties of matter determine the velocity of sound.

*i.e.* velocity of sound is the characteristic of the medium in which wave propagate.

$v = \sqrt{\frac{E}{\dots}}$  ( $E$  = Elasticity of the medium;  $\dots$  = Density of the medium)

#### Important points

(1) As solids are most elastic while gases least *i.e.*  $E_S > E_L > E_G$ . So the velocity of sound is maximum in solids and minimum in gases

$$V_{steel} > V_{water} > V_{air}$$



$$5000 \text{ m/s} > 1500 \text{ m/s} > 330 \text{ m/s}$$

As for sound  $v_{\text{water}} > v_{\text{Air}}$  while for light  $v_w < v_A$ .

Water is rarer than air for sound and denser for light.

The concept of rarer and denser media for a wave is through the velocity of propagation (and not density). Lesser the velocity, denser is said to be the medium and vice-versa.

(2) **Newton's formula** : He assumed that when sound propagates through air temperature remains constant. (*i.e.* the process is isothermal)  $v_{\text{air}} = \sqrt{\frac{K}{\dots}} = \sqrt{\frac{P}{\dots}}$  As  $K = E_s = P$ ;  $E_s =$  Isothermal elasticity;  $P =$  Pressure.

By calculation  $v_{\text{air}} = 279 \text{ m/sec}$ .

However the experimental value of sound in air is 332 m/sec which is greater than that given by Newton's formula.

(3) **Laplace correction** : He modified Newton's formula assuming that propagation of sound in air as adiabatic process.

$$v = \sqrt{\frac{k}{\dots}} = \sqrt{\frac{E_w}{\dots}} \quad (\text{As } k = E_w = \dots = \text{Adiabatic elasticity})$$

$$v = \sqrt{1.41} \times 279 = 331.3 \text{ m/s} \quad (\gamma_{\text{Air}} = 1.41)$$

(4) **Effect of density** :  $v = \sqrt{\frac{\gamma P}{\dots}} \Rightarrow v \propto \frac{1}{\sqrt{\dots}}$

(5) **Effect of pressure** :  $v = \sqrt{\frac{\gamma P}{\dots}} = \sqrt{\frac{\gamma R T}{M}}$ . Velocity of sound is independent of the pressure of gas provided the temperature remains constant. ( $P \propto \dots$  when  $T = \text{constant}$ )

(6) **Effect of temperature** :  $v = \sqrt{\frac{\gamma R T}{M}} \Rightarrow v \propto \sqrt{T(\text{in } K)}$

When the temperature change is small then  $v_t = v_0 (1 + r \Delta t)$

$v_0 =$  velocity of sound at  $0^\circ \text{C}$ ,  $v_t =$  velocity of sound at  $t^\circ \text{C}$ ,  $r =$  temp-coefficient of velocity of sound.

Value of  $r = 0.608 \frac{\text{m/s}}{^\circ \text{C}} = 0.61$  (Approx.)

*Temperature coefficient of velocity* of sound is defined as the change in the velocity of sound, when temperature changes by  $1^\circ\text{C}$ .

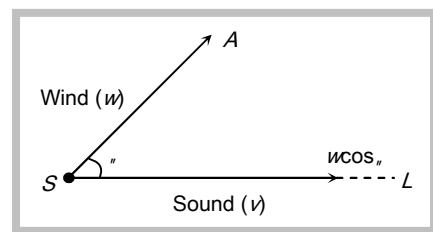
(7) **Effect of humidity** : With increase in humidity, density of air decreases. So with rise in humidity velocity of sound increases.

This is why sound travels faster in humid air (rainy season) than in dry air (summer) at the same temperature.

(8) **Effect of wind velocity** : Because wind drifts the medium (air) along its direction of motion therefore the velocity of sound in a particular direction is the algebraic sum of the velocity of sound and the component of wind velocity in that direction. Resultant velocity of sound along  $SL = v + w \cos \theta$ .

(9) Sound of any frequency or wavelength travels through a given medium with the same velocity.

( $v = \text{constant}$ ) For a given medium velocity remains constant. All other factors like phase, loudness pitch, quality *etc.* have practically no effect on sound velocity.



(10) Relation between velocity of sound and root mean square velocity.

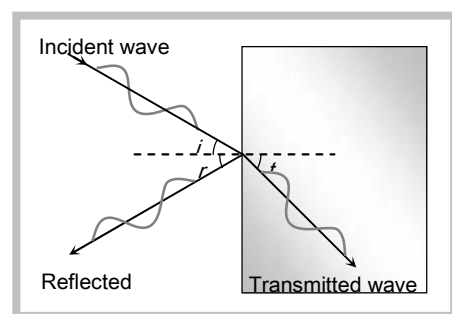
$$v_{\text{sound}} = \sqrt{\frac{\chi RT}{M}} \quad \text{and} \quad v_{\text{rms}} = \sqrt{\frac{3RT}{M}} \quad \text{so} \quad \frac{v_{\text{rms}}}{v_{\text{sound}}} = \sqrt{\frac{3}{\chi}} \quad \text{or} \quad v_{\text{sound}} = [\chi / 3]^{1/2} v_{\text{rms}}$$

(11) There is no atmosphere on moon, therefore propagation of sound is not possible there. To do conversation on moon, the astronaut uses an instrument which can transmit and detect electromagnetic waves.

## 16.6 Reflection and Refraction of Waves

When sound waves are incident on a boundary between two media, a part of incident waves returns back into the initial medium (reflection) while the remaining is partly absorbed and partly transmitted into the second medium (refraction) In case of reflection and refraction of sound

(1) The frequency of the wave remains unchanged that means



$$\tilde{S}_i = \tilde{S}_r = \tilde{S}_t = \tilde{S} = \text{constant}$$

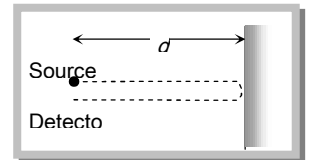
(2) The incident ray, reflected ray, normal and refracted ray all lie in the same plane.

(3) For reflection angle of incidence ( $i$ ) = Angle of reflection ( $r$ )

(4) For refraction  $\frac{\sin i}{\sin r} = \frac{v_i}{v_r}$

(5) In reflection from a denser medium or rigid support, phase changes by  $180^\circ$  and direction reverses if incident wave is  $y = A_i \sin(\tilde{S}t - kx)$  then reflected wave becomes  $y = A_r \sin(\tilde{S}t + kx + f) = -A_r \sin(\tilde{S}t + kx)$ .

(6) In reflection from a rarer medium or free end, phase does not change and direction reverses if incident wave is  $y = A_i \sin(\tilde{S}t - kx)$  then reflected wave becomes  $y = A_r \sin(\tilde{S}t + kx)$



(7) Echo is an example of reflection.

If there is a sound reflector at a distance  $d$  from the source then time interval between original sound and its echo at the site of source will be  $t = \frac{2d}{v}$

## 16.7 Reflection of Mechanical Waves

Medium	Longitudinal wave	Transverse wave	Change in direction	Phase change	Time change	Path change
Reflection from rigid end/denser medium	Compression as rarefaction and vice-versa	Crest as crest and Trough as trough	Reversed	$f$	$\frac{T}{2}$	$\frac{\lambda}{2}$
Reflection from free end/rarer medium	Compression as compression and rarefaction as rarefaction	Crest as trough and trough as crest	No change	Zero	Zero	Zero

## 16.8 Progressive Wave

(1) These waves propagate in the forward direction of medium with a finite velocity.

(2) Energy and momentum are transmitted in the direction of propagation of waves without actual transmission of matter.

(3) In progressive waves, equal changes in pressure and density occurs at all points of medium.

(4) Various forms of progressive wave function.

$$(i) y = A \sin (\check{S} t - kx)$$

where  $y$  = displacement

$A$  = amplitude

$$(ii) y = A \sin \left( \check{S} t - \frac{2f}{\} x \right)$$

$\check{S}$  = angular frequency

$$(iii) y = A \sin 2f \left[ \frac{t}{T} - \frac{x}{\} \right]$$

$n$  = frequency

$k$  = propagation constant

$$(iv) y = A \sin \frac{2f}{\} (vt - x)$$

$T$  = time period

$\}$  = wave length

$$(v) y = A \sin \check{S} \left( t - \frac{x}{v} \right)$$

### Important points

(a) If the sign between  $t$  and  $x$  terms is negative the wave is propagating along positive  $X$ -axis and if the sign is positive then the wave moves in negative  $X$ -axis direction.

(b) The coefficient of sin or cos functions *i.e.* Argument of sin or cos function *i.e.*  $(\check{S} t - kx) = \text{Phase}$ .

(c) The coefficient of  $t$  gives angular frequency  $\check{S} = 2fn = \frac{2f}{T} = vk$ .

(d) The coefficient of  $x$  gives propagation constant or wave number  $k = \frac{2f}{\} = \frac{\check{S}}{v}$ .

(e) The ratio of coefficient of  $t$  to that of  $x$  gives wave or phase velocity. *i.e.*  $v = \frac{\check{S}}{k}$ .

(f) When a given wave passes from one medium to another its frequency does not change.

(g) From  $v = n\} \Rightarrow v \propto \} \therefore n = \text{constant} \Rightarrow \frac{v_1}{v_2} = \frac{\}_1}{\}_2$ .

(5) **Some terms related to progressive waves**

(i) **Wave number ( $\bar{n}$ )**: The number of waves present in unit length is defined as the wave number ( $\bar{n}$ ) =  $\frac{1}{\}$ .

Unit =  $\text{meter}^{-1}$ ; Dimension =  $[L^{-1}]$ .

(ii) **Propagation constant ( $k$ )**:  $k = \frac{w}{x} = \frac{\text{Phase difference between particles}}{\text{Distance between them}}$

$$k = \frac{\check{S}}{v} = \frac{\text{Angular velocity}}{\text{Wave velocity}} \quad \text{and} \quad k = \frac{2f}{\lambda} = 2\pi \check{f}$$

(iii) **Wave velocity ( $v$ )**: The velocity with which the crests and troughs or compression and rarefaction travel in a medium, is defined as wave velocity  $v = \frac{\check{S}}{k} = n\lambda = \frac{\check{S}}{2f} = \frac{\lambda}{T}$ .

(iv) **Phase and phase difference**: Phase of the wave is given by the argument of sine or cosine in the equation of wave. It is represented by  $w(x, t) = \frac{2\pi}{\lambda}(vt - x)$ .

(v) At a given position (for fixed value of  $x$ ) phase changes with time ( $t$ ).

$$\frac{dw}{dt} = \frac{2\pi v}{\lambda} = \frac{2\pi}{T} \Rightarrow dw = \frac{2\pi}{T} dt \Rightarrow \text{Phase difference} = \frac{2\pi}{T} \times \text{Time difference.}$$

(vi) At a given time (for fixed value of  $t$ ) phase changes with position ( $x$ ).

$$\frac{dw}{dx} = \frac{2\pi}{\lambda} \Rightarrow dw = \frac{2\pi}{\lambda} dx \Rightarrow \text{Phase difference} = \frac{2\pi}{\lambda} \times \text{Path difference}$$

$$\Rightarrow \text{Time difference} = \frac{T}{\lambda} \times \text{Path difference}$$

### Sample problems based on Progressive wave

**Problem 1.** The speed of a wave in a certain medium is 960 m/sec. If 3600 waves pass over a certain point of the medium in 1 minute, the wavelength is

- (a) 2 meters                      (b) 4 meters                      (c) 8 meters                      (d) 16 meters

**Solution:** (d)  $v = 960 \text{ m/s}$ ;  $n = \frac{3600}{60} \text{ Hz}$ . So  $\lambda = \frac{v}{n} = \frac{960}{60} = 16 \text{ meters}$ .

**Problem 2.** A simple harmonic progressive wave is represented by the equation  $y = 8 \sin 2\pi (0.1x - 2t)$  where  $x$  and  $y$  are in cm and  $t$  is in seconds. At any instant the Phase difference between two particles separated by 2.0 cm in the  $x$ -direction is

- (a)  $18^\circ$                       (b)  $36^\circ$                       (c)  $54^\circ$                       (d)  $72^\circ$

**Solution:** (d)  $y = 8 \sin 2\pi \left( \frac{x}{10} - 2t \right)$  given by comparing with standard equation  $y = a \sin 2\pi \left( \frac{t}{T} - \frac{x}{\lambda} \right)$

$$\lambda = 10 \text{ cm}$$

$$\text{So Phase Difference} = \frac{2\pi}{\lambda} \times \text{path difference} = \frac{2\pi}{10} \times 2 = \frac{2}{5} \times 180^\circ = 72^\circ$$

## 14 Wave Motion

**Problem 3.** The frequency of sound wave is  $n$  and its velocity is  $v$  if the frequency is increased to  $4n$  the velocity of the wave will be

- (a)  $v$  (b)  $2v$  (c)  $4v$  (d)  $v/4$

**Solution:** (a) Wave velocity does not depend on the frequency. It depends upon the Elasticity and inertia of the medium.

**Problem 4.** The displacement of a particle is given by  $x = 3 \sin(5ft) + 4 \cos(5ft)$  The amplitude of particle is

- (a) 3 (b) 4 (c) 5 (d) 7

**Solution:** (c) Standard equation :  $x = a \sin St + b \cos St$

$$x = \sqrt{a^2 + b^2} \sin(S t + \tan^{-1}(b/a))$$

Given equation  $x = 3 \sin(5ft) + 4 \cos(5ft)$

$$x = \sqrt{9 + 16} \sin(5ft + \tan^{-1} 4/3)$$

$$x = 5 \sin(5ft + \tan^{-1}(4/3))$$

**Problem 5.** The equation of a transverse wave travelling on a rope is given by  $y = 10 \sin f(0.01x - 2.00t)$  where  $y$  and  $x$  are in  $cm$  and  $t$  in seconds. The maximum transverse speed of a particle in the rope is about

- (a)  $63 \text{ cm/sec}$  (b)  $75 \text{ cm/s}$  (c)  $100 \text{ cm/sec}$  (d)  $121 \text{ cm/sec}$

**Solution:** (a) Standard eq. of travelling wave  $y = A \sin(kx - \omega t)$

By comparing with the given equation  $y = 10 \sin(0.01fx - 2ft)$

$$A = 10 \text{ cm}, S = 2f$$

$$\text{Maximum particle velocity} = AS = 2f \times 10 = 63 \text{ cm/sec}$$

**Problem 6.** In a wave motion  $y = a \sin(kx - \omega t)$ ,  $y$  can represent

- (a) Electric Field (b) magnetic field (c) Displacement (d) Pressure

**Solution:** (a,b,c,d)

**Problem 7.** Find the ratio of the speed of sound in nitrogen gas to that of helium gas, at  $300 \text{ K}$  is

- (a)  $\frac{1}{2}$  (b)  $\frac{2}{3}$  (c)  $\sqrt{\frac{3}{5}}$  (d)  $\frac{4}{5}$

**Solution:** (c)  $v = \sqrt{\frac{\gamma RT}{M}}$

$$\frac{v_N}{v_{He}} = \sqrt{\frac{\gamma_{N_2} M_{He}}{\gamma_{He} M_{N_2}}} = \sqrt{\frac{7/5 \cdot 4}{5/3 \cdot 28}} = \sqrt{\frac{3}{5}}$$

**Problem 8.** The displacement  $x$  (in metres) of a particle performing simple harmonic motion is related to time  $t$  (in seconds) as  $x = 0.05 \cos\left(4ft + \frac{f}{4}\right)$ . The frequency of the motion will be

- (a) 0.5 Hz                      (b) 1.0 Hz                      (c) 1.5 Hz                      (d) 2.0 Hz

**Solution:** (d) From the given equation, coefficient of  $t = \check{S} = 4f$

$$\therefore n = \frac{\check{S}}{2f} = \frac{4f}{2f} = 2 \text{ Hz}$$

**Problem 9.** A wave is represented by the equation  $Y = 7 \sin\left(7ft - 0.04fx + \frac{f}{3}\right)$   $x$  is in meters and  $t$  is in seconds.

The speed of the wave is

- (a) 175 m/sec                      (b) 49 f m/s                      (c)  $\frac{49}{f}$  m/s                      (d) 0.28 f m/s

**Solution:** (a) Standard equation  $y = A \sin(\check{S}t - kx + w_0)$

In a given equation  $\check{S} = 7f, k = 0.04f$

$$v = \frac{\check{S}}{k} = \frac{7f}{.04f} = 175 \text{ m/sec}$$

**Problem 10.** A wave is represented by the equation  $y = 0.5 \sin(10t + x)m$ . It is a travelling wave propagating along the  $x$  direction with velocity.

- (a) 10 m/s                      (b) 20 m/s                      (c) 5 m/s                      (d) None of these

**Solution:** (a)  $v = \check{S} / k = 10 / 1 = 10 \text{ m/s}$

**Problem 11.** A transverse progressive wave on a stretched string has a velocity of  $10 \text{ ms}^{-1}$  and a frequency of 100 Hz. The phase difference between two particles of the string which are 2.5 cm apart will be

- (a)  $f/8$                       (b)  $f/4$                       (c)  $3f/8$                       (d)  $f/2$

**Solution:** (d)  $\lambda = v/n = \frac{10}{100} = 0.1 \text{ m} = 10 \text{ cm}$

$$\text{Phase difference} = \frac{2\pi}{\lambda} \times \text{path difference} = \frac{2\pi}{10} \times 2.5 = \frac{\pi}{2}$$

**Problem 12.** In a stationary wave, all particles are

- (a) At rest at the same time twice in every period of oscillation  
 (b) At rest at the same time only once in every period of oscillation  
 (c) Never at rest at the same time  
 (d) Never at rest at all

**Solution:** (a)

**Problem 13.** The path difference between the two waves

$y_1 = a_1 \sin\left(\tilde{S} t - \frac{2fx}{\lambda}\right)$  and  $y_2 = a_2 \cos\left(\tilde{S} t - \frac{2fx}{\lambda} + w\right)$  is

- (a)  $\frac{\lambda}{2f} w$                       (b)  $\frac{\lambda}{2f} \left(w + \frac{f}{2}\right)$                       (c)  $\frac{2f}{\lambda} \left(w - \frac{f}{2}\right)$                       (d)  $\frac{2f}{\lambda} (w)$

**Solution:** (b)  $y_1 = a_1 \sin\left(\tilde{S} t - \frac{2fx}{\lambda}\right)$ ;  $y_2 = a_2 \sin\left(\tilde{S} t - \frac{2fx}{\lambda} + w + \frac{f}{2}\right)$

Phase difference =  $\left(\tilde{S} t - \frac{2fx}{\lambda} + w + \frac{f}{2}\right) - \left(\tilde{S} t - \frac{2fx}{\lambda}\right) = \left(w + \frac{f}{2}\right)$

Path difference =  $\frac{\lambda}{2f} \times \text{Phase difference} = \frac{\lambda}{2f} \left(w + \frac{f}{2}\right)$

**Problem 14.** A plane wave is described by the equation  $y = 3 \cos\left(\frac{x}{4} - 10t - \frac{f}{2}\right)$ . The maximum velocity of the particles of the medium due to this wave is

- (a) 30                                      (b)  $3f/2$                                       (c)  $3/4$                                       (d) 40

**Solution:** (a) Maximum velocity =  $AS = 3 \times 10 = 30$

**Problem 15.** A wave represented by the given equation  $y = A \sin(10fx + 15ft + \frac{f}{3})$  where  $x$  is in meter and  $t$  is in second. The expression represents

- (a) A wave travelling in the positive  $x$ -direction with a velocity of  $1.5 \text{ m/sec}$   
 (b) A wave travelling in the negative  $x$ -direction with a velocity of  $1.5 \text{ m/sec}$   
 (c) A wave travelling in the negative  $x$ -direction with a wavelength of  $0.2 \text{ m}$   
 (d) A wave travelling in the positive  $x$ -direction with a wavelength of  $0.2 \text{ m}$

**Solution:** (b, c) By comparing with standard equation  $Y = A \sin(kx + \tilde{S}t + f/3)$

$$K = 10f, \quad \tilde{S} = 15f$$

We know that :  $v = \frac{\tilde{S}}{k} = 1.5 \text{ m/sec}$ ;  $\lambda = \frac{2f}{k} = 0.2 \text{ meter}$ .

**Problem 16.** A transverse wave is described by the equation  $Y = y_0 \sin 2f\left(ft - \frac{x}{\lambda}\right)$  The maximum particle velocity is four times the wave velocity if

- (a)  $\lambda = \frac{fy_0}{4}$                       (b)  $\lambda = \frac{fy_0}{2}$                       (c)  $\lambda = fy_0$                       (d)  $\lambda = 2fy_0$

**Solution:** (b) Maximum particle velocity = 4 wave velocity

$$AS = 4f\lambda$$

$$y_0 2ff = 4f\lambda$$

$$\lambda = \frac{fy_0}{2}$$



**Problem 17.** The equation of a wave travelling in a string can be written as  $y = 3 \cos f (100 t - x)$  Its wavelength is

- (a) 100 cm                      (b) 2 cm                      (c) 5 cm                      (d) None of these

**Solution:** (b)  $y = A \cos (\omega t - kx)$  – standard equation

$y = 3 \cos (100 f t - f x)$  – given equation

$$\text{So } k = f \text{ and } \lambda = \frac{2\pi}{k} = 2 \text{ cm}$$

**Problem 18.** A plane wave is represented by  $x = 1.2 \sin (314 t + 12.56 y)$  where  $x$  and  $y$  are distances measured along in  $x$  and  $y$  direction in meter and  $t$  is time in seconds. This wave has

- (a) A wave length of 0.25 m and travels in +ve  $x$ -direction  
 (b) A wavelength of 0.25 m and travels in +ve  $y$ -direction  
 (c) A wavelength of 0.5 m and travels in –ve  $y$ -direction  
 (d) A wavelength of 0.5 m and travels in –ve  $x$ -direction

**Solution:** (c) From given equation  $k = 12.56$

$$\lambda = \frac{2\pi}{k} = 0.5 \text{ m} \text{ direction} = -y$$

**Problem 19.** A wave is reflected from a rigid support. The change in phase on reflection will be

- (a)  $\pi/4$                       (b)  $\pi/2$                       (c)  $\pi$                       (d)  $2\pi$

**Solution:** (c)

**Problem 20.** The equation of displacement of two waves are given as  $y_1 = 10 \sin \left( 3\pi t + \frac{\pi}{3} \right)$ ;  $y_2 = 5$

$$[\sin 3\pi t + \sqrt{3} \cos 3\pi t]$$

Then what is the ratio of their amplitudes

- (a) 1 : 2                      (b) 2 : 1                      (c) 1 : 1                      (d) None of these

**Solution:** (c)  $y_2 = 5 [\sin 3\pi t + \sqrt{3} \cos 3\pi t] = 5 \sqrt{1+3} \sin \left( 3\pi t + \frac{\pi}{3} \right) = 10 \sin \left( 3\pi t + \frac{\pi}{3} \right)$

So,  $A_1 = 10$  and  $A_2 = 10$

**Problem 21.** The equation of a wave travelling on a string is  $y = 4 \sin \frac{\pi}{2} \left( 8t - \frac{x}{8} \right)$  if  $x$  and  $y$  are in cm, then velocity of wave is

- (a) 64 cm/sec in –  $x$  direction                      (b) 32 cm/sec in –  $x$  direction  
 (c) 32 cm/sec in +  $x$  direction                      (d) 64 cm/sec in +  $x$  direction

## 18 Wave Motion

**Solution:** (d)  $y = 4 \sin \left( 4\pi t - \frac{\pi}{16} x \right)$

$$\omega = 4\pi, k = \pi / 16$$

$$v = \frac{\omega}{k} = \frac{4\pi}{\pi / 16} = 64 \text{ cm / sec in } +x \text{ direction.}$$

**Problem 22.** The equation of wave is  $y = 2 \sin \pi (0.5x - 200t)$  where  $x$  and  $y$  are expressed in  $cm$  and  $t$  in  $sec$ . The wave velocity is

- (a) 100  $cm/sec$                       (b) 200  $cm/sec$                       (c) 300  $cm/sec$                       (d) 400  $cm/sec$

**Solution:** (d)  $v = \frac{\omega}{k} = \frac{200\pi}{0.5\pi} = 400 \text{ cm/sec}$

### 16.9 Principle of Superposition

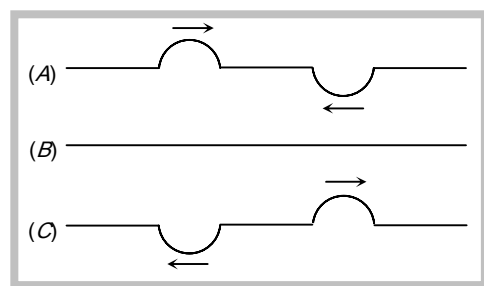
The displacement at any time due to any number of waves meeting simultaneously at a point in a medium is the vector sum of the individual displacements due each one of the waves at that point at the same time.

If  $\vec{y}_1, \vec{y}_2, \vec{y}_3, \dots$  are the displacements at a particular time at a particular position, due to individual waves, then the resultant displacement.  $\vec{y} = \vec{y}_1 + \vec{y}_2 + \vec{y}_3 + \dots$

#### Examples

(i) Radio waves from different stations having different frequencies cross the antenna. But our T.V./Radio set can pick up any desired frequency.

(ii) When two pulses of equal amplitude on a string approach each other [fig. (A)], then on meeting, they superimpose to produce a resultant pulse of zero amplitude [fig (B)]. After crossing, the two pulses travel independently as shown in [fig (C)] as if nothing had happened.



Important applications of superposition principle :

- (a) Interference of waves                      (b) Stationary waves      (c) Beats.

### 16.10 Interference of Sound Waves

When two waves of same frequency, same wavelength, same velocity (nearly equal amplitude) moves in the same direction, Their superimposition results in the interference. Due to interference the resultant intensity

of sound at that point is different from the sum of intensities due to each wave separately. This modification of intensity due to superposition of two or more waves is called interference.

Let at a given point two waves arrive with phase difference  $w$  and the equation of these waves is given by

$y_1 = a_1 \sin \check{S} t$ ,  $y_2 = a_2 \sin (\check{S} t + w)$  then by the principle of superposition

$$\vec{y} = \vec{y}_1 + \vec{y}_2 \Rightarrow y = A \sin (\check{S} t + \alpha) \quad \text{where } A = \sqrt{a_1^2 + a_2^2 + 2a_1a_2 \cos w} \quad \text{and } \tan \alpha = \frac{a_2 \sin w}{a_1 + a_2 \cos w}$$

and since Intensity  $\propto A^2$ .

$$\text{So } I = a_1^2 + a_2^2 + 2a_1a_2 \cos w \Rightarrow I = I_1 + I_2 + 2\sqrt{I_1I_2} \cos w$$

#### Important points

(1) **Constructive interference** : Intensity will be maximum

when  $w = 0, 2f, 4f, \dots, 2fn$  ; where  $n = 0, 1, 2, \dots$

when  $x = 0, \lambda, 2\lambda, \dots, n\lambda$ ; where  $n = 0, 1, \dots$

$$I_{max} = I_1 + I_2 + 2\sqrt{I_1I_2} = (\sqrt{I_1} + \sqrt{I_2})^2 \propto (A_1 + A_2)^2$$

It means the intensity will be maximum at those points where path difference is an integral multiple of wavelength  $\lambda$ . These points are called points of constructive interference or interference maxima.

(2) **Destructive interference** : Intensity will be minimum

when  $w = f, 3f, 5f, \dots, (2n-1)f$  ; where  $n = 1, 2, 3, \dots$

when  $x = \lambda/2, 3\lambda/2, \dots, (2n-1)\lambda/2$ ; where  $n = 1, 2, 3, \dots$

$$I_{min} = I_1 + I_2 - 2\sqrt{I_1I_2} \Rightarrow I_{min} = (\sqrt{I_1} - \sqrt{I_2})^2 \propto (A_1 - A_2)^2$$

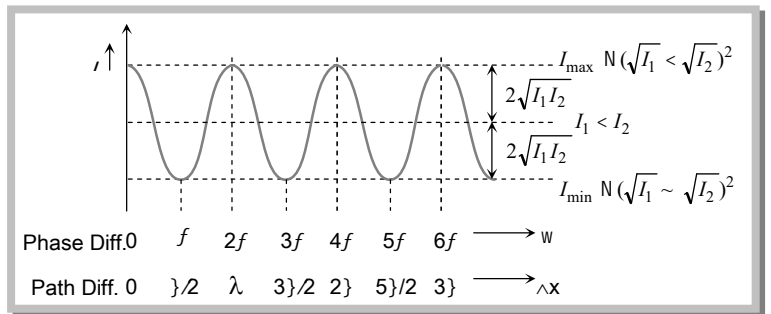
(3) All maxima are equally spaced and equally loud. Same is also true for minima. Also interference

maxima and minima are alternate as for maximum  $\Delta x = 0, \lambda, 2\lambda, \dots$  etc. and for minimum  $\Delta x = \frac{\lambda}{2}, \frac{3\lambda}{2}, \frac{5\lambda}{2}, \dots$  etc .

$$(4) \frac{I_{\max}}{I_{\min}} = \frac{(\sqrt{I_1} + \sqrt{I_2})^2}{(\sqrt{I_1} - \sqrt{I_2})^2} = \frac{(A_1 + A_2)^2}{(A_1 - A_2)^2} \text{ with } \frac{I_1}{I_2} = \frac{A_1^2}{A_2^2}$$

(5) If  $I_1 = I_2 = I_0$  then  $I_{\max} = 4I_0$  and  $I_{\min} = 0$

(6) In interference the intensity in maximum  $(\sqrt{I_1} + \sqrt{I_2})^2$  exceeds the sum of individual intensities  $(I_1 + I_2)$  by an amount  $2\sqrt{I_1 I_2}$  while in minima  $(\sqrt{I_1} - \sqrt{I_2})^2$  lacks  $(I_1 + I_2)$  by the same amount  $2\sqrt{I_1 I_2}$ .



Hence in interference energy is neither created nor destroyed but is redistributed.

### 16.11 Standing Waves or Stationary Waves

When two sets of progressive wave trains of same type (both longitudinal or both transverse) having the same amplitude and same time period/frequency/wavelength travelling with same speed along the same straight line in opposite directions superimpose, a new set of waves are formed. These are called stationary waves or standing waves.

Characteristics of standing waves :

(1) The disturbance confined to a particular region between the starting point and reflecting point of the wave.

(2) There is no forward motion of the disturbance from one particle to the adjoining particle and so on, beyond this particular region.

(3) The total energy associated with a stationary wave is twice the energy of each of incident and reflected wave. But there is no flow or transference of energy along the stationary wave.

(4) There are certain points in the medium in a standing wave, which are permanently at rest. These are called nodes. The distance between two consecutive nodes is  $\frac{\lambda}{2}$ .

(5) Points of maximum amplitude is known as antinodes. The distance between two consecutive antinodes is also  $\frac{\lambda}{2}$ . The distance between a node and adjoining antinode is  $\frac{\lambda}{4}$ .

(6) The medium splits up into a number of segments. Each segment is vibrating up and down as a whole.

(7) All the particles in one particular segment vibrate in the same phase. Particles in two consecutive segments differ in phase by  $180^\circ$ .

(8) All the particles except those at nodes, execute simple harmonic motion about their mean position with the same time period.

(9) The amplitude of vibration of particles varies from zero at nodes to maximum at antinodes.

(10) Twice during each vibration, all the particles of the medium pass simultaneously through their mean position.

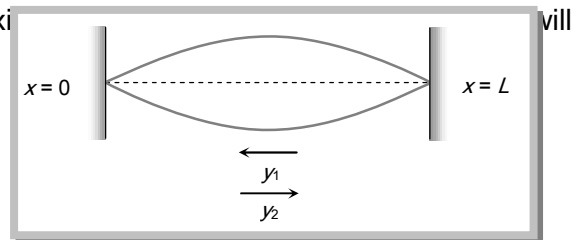
(11) The wavelength and time period of stationary waves are the same as for the component waves.

(12) Velocity of particles while crossing mean position varies from maximum at antinodes to zero at nodes.

(13) In standing waves, if amplitude of component waves are not equal. Resultant amplitude at nodes will be minimum (but not zero). Therefore, some energy will pass across nodes and waves will be partially standing.

### 16.12 Standing Waves on a String

When a string under tension is set into vibration, transverse harmonic waves propagate along its length. When the length of string is fixed, reflected waves will also exist. These waves will superimpose to produce transverse stationary waves in a string



$$\text{Incident wave } y_1 = a \sin \frac{2f}{\lambda} (vt + x)$$

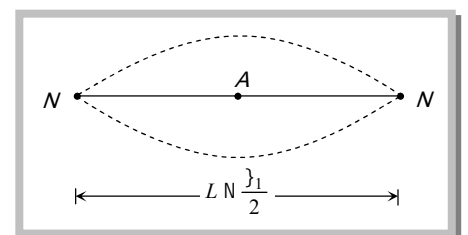
$$\text{Reflected wave } y_2 = a \sin \frac{2f}{\lambda} [(vt - x) + f] = -a \sin \frac{2f}{\lambda} (vt - x)$$

$$\text{According to superposition principle : } y = y_1 + y_2 = 2a \cos \frac{2fvt}{\lambda} \sin \frac{2fx}{\lambda}$$

General formula for wavelength  $\lambda = \frac{2L}{n}$  where  $n = 1, 2, 3, \dots$  correspond to 1<sup>st</sup>, 2<sup>nd</sup>, 3<sup>rd</sup> modes of vibration of the string.

$$(1) \text{ First normal mode of vibration } n_1 = \frac{v}{\lambda_1} = \frac{v}{2L} \Rightarrow n_1 = \frac{1}{2L} \sqrt{\frac{T}{m}}$$

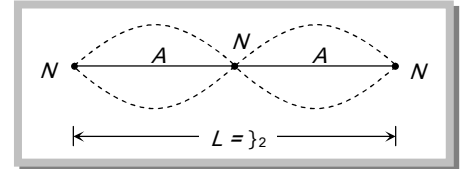
This mode of vibration is called the fundamental mode and the frequency is called fundamental frequency. The sound from the note so



produced is called fundamental note or first harmonic.

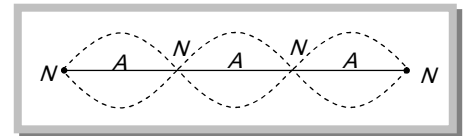
(2) Second normal mode of vibration :  $n_2 = \frac{v}{\lambda_2} = \frac{v}{L} = \frac{2v}{2L} = 2(n_1)$

This is second harmonic or first overtone.



(3) Third normal mode of vibration :  $n_3 = \frac{v}{\lambda_3} = \frac{3v}{2L} = 3n_1$

This is third harmonic or second overtone.



**Position of nodes :**  $x = 0, \frac{L}{n}, \frac{2L}{n}, \frac{3L}{n} \dots \dots \dots L$

For first mode of vibration  $x = 0, x = L$  [Two nodes]

For second mode of vibration  $x = 0, x = \frac{L}{2}, x = L$  [Three nodes]

For third mode of vibration  $x = 0, x = \frac{L}{3}, x = \frac{2L}{3}, x = L$  [Four nodes]

**Position of antinodes :**  $x = \frac{L}{2n}, \frac{3L}{2n}, \frac{5L}{2n} \dots \dots \dots \frac{(2y - 1)L}{2n}$

For first mode of vibration  $x = L/2$  [One antinode]

For second mode of vibration  $x = \frac{L}{4}, \frac{3L}{4}$  [Two antinode]

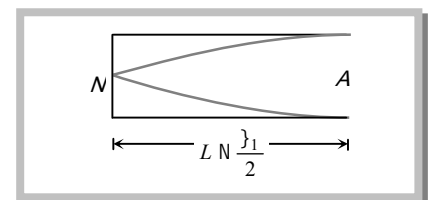
**16.13 Standing Wave in a Closed Organ Pipe**

Organ pipes are the musical instrument which are used for producing musical sound by blowing air into the pipe. Longitudinal stationary waves are formed on account of superimposition of incident and reflected longitudinal waves.

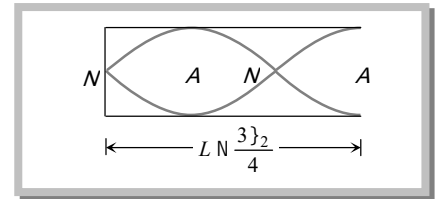
Equation of standing wave  $y = 2a \cos \frac{2fv t}{\lambda} \sin \frac{2fx}{\lambda}$

General formula for wavelength  $\lambda = \frac{4L}{(2n - 1)}$

(1) First normal mode of vibration :  $n_1 = \frac{v}{4L}$



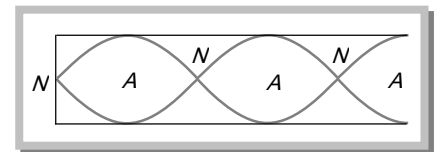
This is called fundamental frequency. The note so produced is called fundamental note or first harmonic.



(2) Second normal mode of vibration :  $n_2 = \frac{v}{\lambda_2} = \frac{3v}{4L} = 3 n_1$

This is called *third harmonic* or *first overtone*.

(3) Third normal mode of vibration :  $n_3 = \frac{5v}{4L} = 5 n_1$



This is called *fifth harmonic* or *second overtone*.

**Position of nodes :**  $x = 0, \frac{2L}{(2n-1)}, \frac{4L}{(2n-1)}, \frac{6L}{(2n-1)}, \dots, \frac{2nL}{(2n-1)}$

For first mode of vibration  $x = 0$  [One node]

For second mode of vibration  $x = 0, x = \frac{2L}{3}$  [Two nodes]

For third mode of vibration  $x = 0, x = \frac{2L}{5}, \frac{4L}{5}$  [Three nodes]

**Position of antinode :**  $x = \frac{L}{2n-1}, \frac{3L}{2n-1}, \frac{5L}{2n-1}, \dots, L$

For first mode of vibration  $x = L$  [One antinode]

For second mode of vibration  $x = \frac{L}{3}, x = L$  [Two antinode]

For third mode of vibration  $x = \frac{L}{5}, \frac{3L}{5}, L$  [Three antinode]

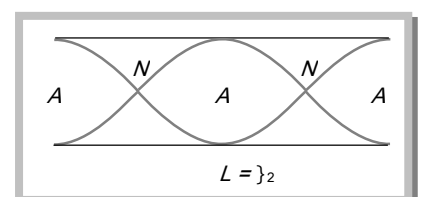
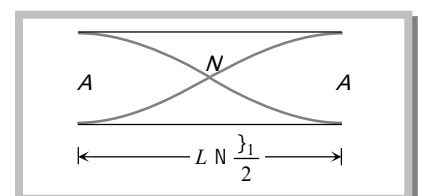
### 16.14 Standing Waves in Open Organ Pipes

General formula for wavelength

$\lambda = \frac{2L}{n}$  where  $n = 1, 2, 3, \dots$

(1) First normal mode of vibration :  $n_1 = \frac{v}{\lambda_1} = \frac{v}{2L}$

This is called fundamental frequency and the note so produced is called *fundamental note* or *first harmonic*.

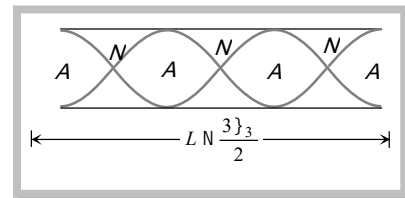


(2) Second normal mode of vibration  $n_2 = \frac{v}{\lambda_2} = \frac{v}{L} = 2\left(\frac{v}{2L}\right) = 2n_1 \Rightarrow n_2 = 2n_1$

This is called *second harmonic* or *first overtone*.

(3) Third normal mode of vibration  $n_3 = \frac{v}{\lambda_3} = \frac{3v}{2L}$ ,  $n_3 = 3n_1$

This is called *third harmonic* or *second overtone*.



### Important points

(i) Comparison of closed and open organ pipes shows that fundamental note in open organ pipe  $\left(n_1 = \frac{v}{2L}\right)$  has double the frequency of the fundamental note in closed organ pipe  $\left(n_1 = \frac{v}{4L}\right)$ .

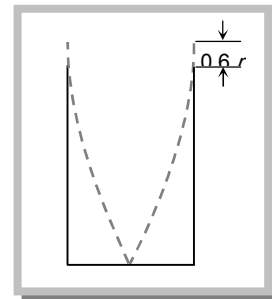
Further in an open organ pipe all harmonics are present whereas in a closed organ pipe, only alternate harmonics of frequencies  $n_1, 3n_1, 5n_1, \dots$  etc are present. The harmonics of frequencies  $2n_1, 4n_1, 6n_1, \dots$  are missing.

Hence musical sound produced by an open organ pipe is sweeter than that produced by a closed organ pipe.

(ii) Harmonics are the notes/sounds of frequency equal to or an integral multiple of fundamental frequency ( $n$ ). Thus the first, second, third, harmonics have frequencies  $n_1, 2n_1, 3n_1, \dots$

(iii) Overtones are the notes/sounds of frequency twice/thrice/ four times the fundamental frequency ( $n$ ) eg.  $2n, 3n, 4n, \dots$  and so on.

(iv) In organ pipe an antinode is not formed exactly at the open end rather it is formed a little distance away from the open end outside it. The distance of antinode from the open end of the pipe is known as end correction.



### 16.15 Vibration of a String

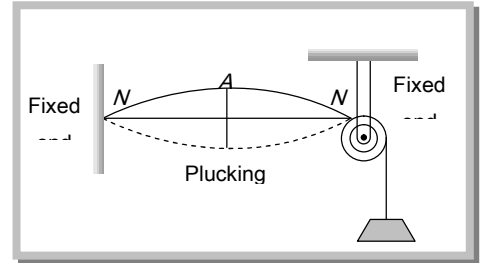
Fundamental frequency  $n = \frac{1}{2L} \sqrt{\frac{T}{m}}$



General formula  $n_p = \frac{p}{2L} \sqrt{\frac{T}{m}}$

$L$  = Length of string,  $T$  = Tension in the string

$m$  = Mass per unit length (linear density),  $p$  = mode of vibration



### Important points

(1) As a string has many natural frequencies, so when it is excited with a tuning fork, the string will be in resonance with the given body if any of its natural frequencies coincides with the body.

(2) (i)  $n \propto \frac{1}{L}$  if  $T$  and  $m$  are constant (ii)  $n \propto \sqrt{T}$  if  $L$  and  $m$  are constant (iii)  $n \propto \frac{1}{\sqrt{m}}$  if  $T$  and  $L$  are constant

(3) If  $M$  is the mass of the string of length  $L$ ,  $m = \frac{M}{L}$

So  $n = \frac{1}{2L} \sqrt{\frac{T}{m}} = \frac{1}{2L} \sqrt{\frac{T}{M/L}} = \frac{1}{2} \sqrt{\frac{T}{ML}} = \frac{1}{2L} \sqrt{\frac{T}{fr^2 \dots}} = \frac{1}{2Lr} \sqrt{\frac{T}{f \dots}}$  where  $m = fr^2 \dots$  ( $r$  = Radius,  $\dots$  = Density)

## 16.16 Comparative Study of Stretched Strings, Open Organ Pipe and Closed Organ Pipe

S. No.	Parameter	Stretched string	Open organ pipe	Closed organ pipe
(1)	Fundamental frequency or 1 <sup>st</sup> harmonic	$n_1 = \frac{v}{2l}$	$n_1 = \frac{v}{2l}$	$n_1 = \frac{v}{4l}$
(2)	Frequency of 1 <sup>st</sup> overtone or 2 <sup>nd</sup> harmonic	$n_2 = 2n_1$	$n_2 = 2n_1$	Missing
(3)	Frequency of 2 <sup>nd</sup> overtone or 3 <sup>rd</sup> harmonic	$n_3 = 3n_1$	$n_3 = 3n_1$	$n_3 = 3n_1$
(4)	Frequency ratio of overtones	2 : 3 : 4...	2 : 3 : 4...	3 : 5 : 7...
(5)	Frequency ratio of harmonics	1 : 2 : 3 : 4...	1 : 2 : 3 : 4...	1 : 3 : 5 : 7...
(6)	Nature of waves	Transverse stationary	Longitudinal stationary	Longitudinal stationary

### 16.17 Beats

When two sound waves of slightly different frequencies, travelling in a medium along the same direction, superimpose on each other, the intensity of the resultant sound at a particular position rises and falls regularly with time. This phenomenon of regular variation in intensity of sound with time at a particular position is called beats.

#### Important points

(1) **One beat** : If the intensity of sound is maximum at time  $t = 0$ , one beat is said to be formed when intensity becomes maximum again after becoming minimum once in between.

(2) **Beat period** : The time interval between two successive beats (*i.e.* two successive maxima of sound) is called beat period.

(3) **Beat frequency** : The number of beats produced per second is called beat frequency.

(4) **Persistence of hearing** : The impression of sound heard by our ears persist in our mind for  $1/10^{\text{th}}$  of a second. If another sound is heard before  $1/10$  second is over, the impression of the two sound mix up and our mind cannot distinguish between the two.

So for the formation of distinct beats, frequencies of two sources of sound should be nearly equal (difference of frequencies less than 10)

(5) **Equation of beats** : If two waves of equal amplitudes ' $a$ ' and slightly different frequencies  $n_1$  and  $n_2$  travelling in a medium in the same direction are.

$$y_1 = a \sin \tilde{S}_1 t = a \sin 2\pi n_1 t; \quad y_2 = a \sin \tilde{S}_2 t = a \sin 2\pi n_2 t$$

By the principle of super position :  $\vec{y} = \vec{y}_1 + \vec{y}_2$

$$y = A \sin f (n_1 + n_2)t \quad \text{where } A = 2a \cos f (n_1 - n_2)t = \text{Amplitude of resultant wave.}$$

(6) **Beat frequency** :  $n = n_1 \sim n_2$ .

(7) **Beat period** :  $T = \frac{1}{\text{Beat frequency}} = \frac{1}{n_1 \sim n_2}$

### 16.18 Determination of Unknown Frequency

Let  $n_2$  is the unknown frequency of tuning fork  $B$ , and this tuning fork  $B$  produce  $x$  beats per second with another tuning fork of known frequency  $n_1$ .

As number of beat/sec is equal to the difference in frequencies of two sources, therefore  $n_2 = n_1 \pm x$

The positive/negative sign of  $x$  can be decided in the following two ways :

By loading	By filing
<b>If B is loaded with wax so its frequency decreases</b>	<b>If B is filed, its frequency increases</b>
If number of beats decreases $n_2 = n_1 + x$	If number of beats decreases $n_2 = n_1 - x$
If number of beats Increases $n_2 = n_1 - x$	If number of beats Increases $n_2 = n_1 + x$
If number of beats remains unchanged $n_2 = n_1 + x$	If number of beats remains unchanged $n_2 = n_1 - x$
If number of beats becomes zero $n_2 = n_1 + x$	If number of beats becomes zero $n_2 = n_1 - x$
<b>If A is loaded with wax its frequency decreases</b>	<b>If A is filed, its frequency increases</b>
If number of beats decreases $n_2 = n_1 - x$	If number of beats decreases $n_2 = n_1 + x$
If number of beats increases $n_2 = n_1 + x$	If number of beats Increases $n_2 = n_1 - x$
If number of beats remains unchanged $n_2 = n_1 - x$	If number of beats remains unchanged $n_2 = n_1 + x$
If number of beats becomes zero $n_2 = n_1 - x$	If no of beats becomes zero $n_2 = n_1 + x$

### Sample problems based on Superposition of waves

- Problem 23.** The stationary wave produced on a string is represented by the equation  $y = 5 \cos\left(\frac{fx}{3}\right) \sin(40ft)$  where  $x$  and  $y$  are in  $cm$  and  $t$  is in seconds. The distance between consecutive nodes is
- (a)  $5\text{ cm}$                       (b)  $f\text{ cm}$                       (c)  $3\text{ cm}$                       (d)  $40\text{ cm}$

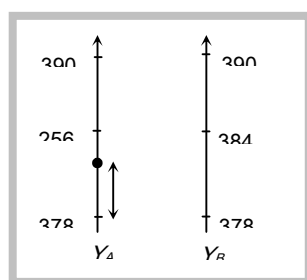
**Solution:** (c) By comparing with standard equation of stationary wave

$$y = a \cos \frac{2fx}{\lambda} \sin \frac{2fvt}{\lambda}$$

We get  $\frac{2fx}{\lambda} = \frac{fx}{3} \Rightarrow \lambda = 6$ ; Distance between two consecutive nodes =  $\frac{\lambda}{2} = 3\text{ cm}$

- Problem 24.** On sounding tuning fork A with another tuning fork B of frequency  $384\text{ Hz}$ , 6 beats are produced per second. After loading the prongs of A with wax and then sounding it again with B, 4 Beats are produced per second what is the frequency of the tuning fork A.
- (a)  $388\text{ Hz}$                       (b)  $80\text{ Hz}$                       (c)  $378\text{ Hz}$                       (d)  $390\text{ Hz}$

**Solution:** (c)



Probable frequency of A is 390 Hz and 378 Hz and After loading the beats are decreasing from 6 to 4 so the original frequency of A will be  $n_2 = n_1 - x = 378$  Hz.

**Problem 25.** Two sound waves of slightly different frequencies propagating in the same direction produces beats due to

- (a) Interference                      (b) Diffraction                      (c) Polarization                      (d) Refraction

*Solution:* (a)

**Problem 26.** Beats are produced with the help of two sound waves on amplitude 3 and 5 units. The ratio of maximum to minimum intensity in the beats is

- (a) 2 : 1                                  (b) 5 : 3                                  (c) 4 : 1                                  (d) 16 : 1

*Solution:* (d)  $\frac{I_{\max}}{I_{\min}} = \left(\frac{A_1 + A_2}{A_1 - A_2}\right)^2 = \left(\frac{5 + 3}{5 - 3}\right)^2 = 16:1$

**Problem 27.** Two tuning forks have frequencies 380 and 384 hertz respectively. When they are sounded together, they produce 4 beats. After hearing the maximum sound, how long will it take to hear the minimum sound

- (a) 1/2 sec                                  (b) 1/4 sec                                  (c) 1/8 sec                                  (d) 1/16 sec

*Solution:* (c) Beats period = Time interval between two minima

$$T = \frac{1}{n_1 - n_2} = \frac{1}{4} \text{ sec}$$

Time interval between maximum sound and minimum sound =  $T/2 = 1/8$  sec

**Problem 28.** Two tuning fork A and B give 4 beats per second when sounded together. The frequency of A is 320 Hz. When some wax is added to B and it is sounded with A, 4 beats per second are again heard. The frequency of B is

- (a) 312 Hz                                  (b) 316 Hz                                  (c) 324 Hz                                  (d) 328 Hz

*Solution:* (c) Since there is no change in beats. Therefore the original frequency of B is

$$n_2 = n_1 + x = 320 + 4 = 324$$

**Problem 29.** 41 forks are so arranged that each produces 5 beat/sec when sounded with its near fork. If the frequency of last fork is double the frequency of first fork, then the frequencies of the first and last fork respectively

- (a) 200, 400                      (b) 205, 410                      (c) 195, 390                      (d) 100, 200

**Solution:** (a) Let the frequency of first tuning fork =  $n$  and that of last =  $2n$

$n, n + 5, n + 10, n + 15 \dots \dots 2n$  this forms A.P.

Formula of A.P  $l = a + (N - 1) r$  where  $l =$  Last term,  $a =$  First term,  $N =$  Number of term,  $r =$  Common difference

$$2n = n + (41 - 1) 5$$

$$2n = n + 200$$

$$n = 200 \quad \text{and} \quad 2n = 400$$

**Problem 30.** In stationary waves, antinodes are the points where there is

- (a) Minimum displacement and minimum pressure change  
 (b) Minimum displacement and maximum pressure change  
 (c) Maximum displacement and maximum pressure change  
 (d) Maximum displacement and minimum pressure change

**Solution:** (d) At Antinodes displacement is maximum but pressure change is minimum.

**Problem 31.** The equation  $y = 0.15 \sin 5x \cos 300 t$ , describes a stationary wave. The wavelength of the stationary wave is

- (a) Zero meter                      (b) 1.256 meter                      (c) 2.512 meter                      (d) 0.628 meter

**Solution:** (b) By comparing with standard equation  $\therefore \frac{2fx}{\lambda} = 5x \Rightarrow \lambda = \frac{2}{5} \times f = 1.256 \text{ meter}$

**Problem 32.** The equation of a stationary wave is  $y = 0.8 \cos\left(\frac{fx}{20}\right) \sin 200ft$  where  $x$  is in  $cm$ . and  $t$  is in sec. The separation between consecutive nodes will be

- (a) 20  $cm$                       (b) 10  $cm$                       (c) 40  $cm$                       (d) 30  $cm$

**Solution:** (a) Standard equation  $y = A \cos \frac{2fx}{\lambda} \sin \frac{2fvt}{\lambda}$

By comparing this equation with given equation.  $\frac{2fx}{\lambda} = \frac{fx}{20} \Rightarrow \lambda = 40 \text{ cm}$

Distance Between two nodes =  $\frac{\lambda}{2} = 20 \text{ cm}$ .

**Problem 33.** Which of the property makes difference between progressive and stationary waves

- (a) Amplitude                      (b) Frequency                      (c) Propagation of energy                      (d) Phase of the wave

**Solution:** (c) In stationary waves there is no transfer of energy.

**Problem 34.** If amplitude of waves at distance  $r$  from a point source is  $A$ , the amplitude at a distance  $2r$  will be

- (a)  $2A$                                       (b)  $A$                                       (c)  $A/2$                                       (d)  $A/4$

**Solution:** (c)  $I \propto A^2$  and  $I \propto \frac{1}{r^2}$                       so  $r \propto \frac{1}{A}$ ;                       $\frac{r_1}{r_2} = \frac{A_2}{A_1} \Rightarrow A_2 = A_1 \left( \frac{r_1}{r_2} \right) = A \left( \frac{1}{2} \right) = A/2$

**Problem 35.** If two waves of same frequency and same amplitude respectively on superimposition produced a resultant disturbance of the same amplitude the wave differ in phase by

- (a)  $f$                                       (b)  $2f/3$                                       (c)  $f/2$                                       (d) zero

**Solution:** (b)  $A = \sqrt{A_1^2 + A_2^2 + 2A_1A_2 \cos w}$

$$A^2 = A^2 + A^2 + 2A^2 \cos w \quad [A_1 = A_2 = A \text{ given}]$$

$$\cos w = -1/2 \Rightarrow w = 120^\circ = \frac{2f}{3}$$

**Problem 36.** The superposition takes place between two waves of frequency  $f$  and amplitude  $a$ . The total intensity is directly proportional to

- (a)  $a$                                       (b)  $2a$                                       (c)  $2a^2$                                       (d)  $4a^2$

**Solution:** (d)  $I \propto (a_1 + a_2)^2$                       [As  $a_1 = a_2 = a$ ]

$$I \propto 4a^2$$

**Problem 37.** The following equation represent progressive transverse waves

$$z_1 = A \cos (\tilde{S} t - kx)$$

$$z_2 = A \cos (\tilde{S} t + kx)$$

$$z_3 = A \cos (\tilde{S} t + ky)$$

$$z_4 = A \cos (2\pi t - 2ky)$$

A stationary wave will be formed by superposing

- (a)  $z_1$  and  $z_2$                       (b)  $z_1$  and  $z_4$                       (c)  $z_2$  and  $z_3$                       (d)  $z_3$  and  $z_4$

**Solution:** (a) The direction of wave must be opposite and frequencies will be same then by superposition, standing wave formation takes place.

**Problem 38.** When two sound waves with a phase difference of  $\pi/2$  and each having amplitude  $A$  and frequency  $\omega$  are superimposed on each other, then the maximum amplitude and frequency of resultant wave is

- (a)  $\frac{A}{\sqrt{2}}; \omega/2$                       (b)  $\frac{A}{\sqrt{2}}; \omega$                       (c)  $\sqrt{2}A; \frac{\omega}{2}$                       (d)  $\sqrt{2}A; \omega$

**Solution:** (d) Resultant Amplitude =  $\sqrt{a_1^2 + a_2^2 + 2a_1a_2 \cos W} = \sqrt{A^2 + A^2 + 2A^2 \cos \frac{\pi}{2}} = \sqrt{2}A$

and frequency remains same =  $\omega$ .

**Problem 39.** There is a destructive interference between the two waves of wavelength  $\lambda$  coming from two different paths at a point. To get maximum sound or constructive interference at that point, the path of one wave is to be increased by

- (a)  $\lambda/4$                       (b)  $\lambda/2$                       (c)  $\frac{3\lambda}{4}$                       (d)  $\lambda$

**Solution:** (b) Destructive interference means the path difference is  $(2n-1)\frac{\lambda}{2}$

If it is increased by  $\lambda/2$

Then new path difference  $(2n-1)\frac{\lambda}{2} + \frac{\lambda}{2} = n\lambda$

which is the condition of constructive interference.

**Problem 40.** The tuning fork and sonometer wire were sounded together and produce 4 beats/second when the length of sonometer wire is 95 cm or 100 cm. The frequency of tuning fork is

- (a) 156 Hz                      (b) 152 Hz                      (c) 148 Hz                      (d) 160 Hz

**Solution:** (a) Frequency  $n \propto \frac{1}{l}$                        $\therefore$  As  $n = \frac{1}{2l} \sqrt{\frac{T}{m}}$

If  $n$  is the frequency of tuning fork.

$$n + 4 \propto \frac{1}{95} \Rightarrow n - 4 \propto \frac{1}{100} \Rightarrow (n + 4) 95 = (n - 4) 100 \Rightarrow n = 156 \text{ Hz.}$$

**Problem 41.** A tuning fork  $F_1$  has a frequency of 256 Hz and it is observed to produce 6 beats/second with another tuning fork  $F_2$ . When  $F_2$  is loaded with wax. It still produces 6 beats/second with  $F_1$ . The frequency of  $F_2$  before loading was

- (a) 253 Hz                      (b) 262 Hz                      (c) 250 Hz                      (d) 259 Hz

**Solution:** (b) No of beats does not change even after loading then  $n_2 = n_1 + x = 256 + 6 = 262 \text{ Hz.}$

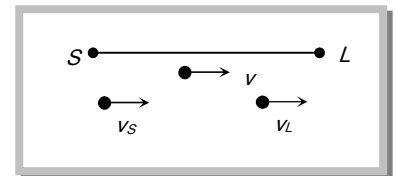
### 16.19 Doppler Effect

Whenever there is a relative motion between a source of sound and the listener, the apparent frequency of sound heard by the listener is different from the actual frequency of sound emitted by the source.

When the distance between the source and listener is decreasing the apparent frequency increases. It means the apparent frequency is more than the actual frequency of sound. The reverse is also true.

$$\text{General expression for apparent frequency } n' = \frac{[(v + v_m) - v_L]n}{[(v + v_m) - v_S]}$$

Here  $n$  = Actual frequency;  $v_L$  = Velocity of listener;  $v_S$  = Velocity of source



$v_m$  = Velocity of medium and  $v$  = Velocity of sound wave

Sign convention : All velocities along the direction  $S$  to  $L$  are taken as positive and all velocities along the direction  $L$  to  $S$  are taken as negative. If the medium is stationary  $v_m = 0$  then  $n' = \left( \frac{v - v_L}{v - v_S} \right) n$

#### Special cases :

(1) Source is moving towards the listener, but the listener at rest  $n' = \frac{v}{v - v_S} .n$

(2) Source is moving away from the listener but the listener is at rest  $n' = \frac{v}{v + v_S} .n$

(3) Source is at rest and listener is moving away from the source  $n' = \frac{v - v_L}{v} n$



(4) Source is at rest and listener is moving towards the source  $n' = \frac{v + v_L}{v} \cdot n$

(5) Source and listener are approaching each other  $n' = \left( \frac{v + v_L}{v - v_S} \right) n$

(6) Source and listener moving away from each other  $n' = \left( \frac{v - v_L}{v + v_S} \right) n$

(7) Both moves in the same direction with same velocity  $n' = n$ , *i.e.* there will be no Doppler effect because relative motion between source and listener is zero.

(8) Source and listener moves at right angle to the direction of wave propagation.  $n' = n$

It means there is no change in frequency of sound heard if there is a small displacement of source and listener at right angle to the direction of wave propagation but for a large displacement the frequency decreases because the distance between source of sound and listener increases.

#### Important points

(i) If the velocity of source and listener is equal to or greater than the sound velocity then Doppler effect is not seen.

(ii) Doppler effect gives information regarding the change in frequency only. It does not says about intensity of sound.

(iii) Doppler effect in sound is asymmetric but in light it is symmetric.

### 16.20 Some Typical Features of Doppler's Effect in Sound

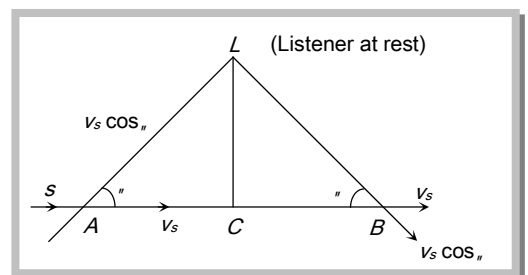
(1) **When a source is moving in a direction making an angle  $\theta$  w.r.t. the listener** : The apparent frequency heard by listener  $L$  at rest

When source is at point  $A$  is  $n' = \frac{nv}{v - v_S \cos \theta}$

As source moves along  $AB$ , value of  $\theta$  increases,  $\cos \theta$  decreases,  $n'$  goes on decreasing.

At point  $C$ ,  $\theta = 90^\circ$ ,  $\cos \theta = \cos 90^\circ = 0$ ,  $n' = n$ .

At point  $B$ , the apparent frequency of sound becomes  $n'' = \frac{nv}{v + v_S \cos \theta}$



(2) **When a source of sound approaches a high wall or a hill with a constant velocity  $v_s$** , the reflected sound propagates in a direction opposite to that of direct sound. We can assume that the source and observer are approaching each other with same velocity *i.e.*  $v_s = v_L$

$$\therefore n' = \left( \frac{v + v_L}{v - v_s} \right) n$$

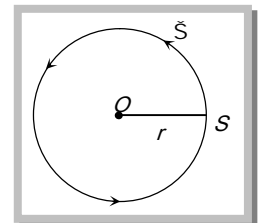
(3) When a listener moves between two distant sound sources : Let  $v_L$  be the velocity of listener away from  $S_1$  and towards  $S_2$ . Apparent frequency from  $S_1$  is  $n' = \frac{(v - v_L)n}{v}$

and apparent frequency heard from  $S_2$  is  $n'' = \frac{(v + v_L)n}{v}$

$$\therefore \text{Beat frequency} = n'' - n' = \frac{2nv_L}{v}$$

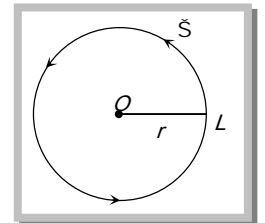
(4) When source is revolving in a circle and listener  $L$  is on one side

$$v_s = r\checkmark \quad \text{so } n_{\max} = \frac{nv}{v - v_s} \quad \text{and } n_{\min} = \frac{nv}{v + v_s}$$



(5) When listener  $L$  is moving in a circle and the source is on one side

$$v_L = r\checkmark \quad \text{so } n_{\max} = \frac{(v + v_L)n}{v} \quad \text{and } n_{\min} = \frac{(v - v_L)n}{v}$$



(6) There will be no change in frequency of sound heard, if the source is situated at the centre of the circle along which listener is moving.

(7) Conditions for no Doppler effect : (i) When source ( $S$ ) and listener ( $L$ ) both are at rest.

(ii) When medium alone is moving.

(iii) When  $S$  and  $L$  move in such a way that distance between  $S$  and  $L$  remains constant.

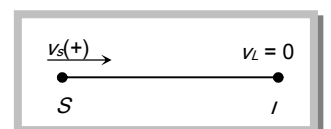
(iv) When source  $S$  and listener  $L$ , are moving in mutually perpendicular directions.

### Sample problems based on Doppler effect

**Problem 42.** A source of sound of frequency 90 vibration/sec is approaching a stationary observer with a speed equal to 1/10 the speed of sound. What will be the frequency heard by the observer

- (a) 80 vibration/sec      (b) 90 vibration/sec      (c) 100 vibration/sec      (d) 120 vibration/sec


**Solution:** (c)  $n' = \frac{v}{v - v_s} \cdot n \Rightarrow n' = \frac{v}{v - \frac{v}{10}} \cdot n \Rightarrow n' = \frac{10}{9} n = \frac{10 \times 90}{9} = 100$  vibration/sec



**Problem 43.** A source of sound of frequency 500 Hz is moving towards an observer with velocity 30 m/s. The speed of the sound is 330 m/s. The frequency heard by the observer will be

- (a) 550 Hz                      (b) 458.3 Hz                      (c) 530 Hz                      (d) 545.5 Hz

**Solution:** (a) 
$$n' = \frac{v}{v - v_s} \cdot n \Rightarrow n' = \frac{330}{330 - 30} \cdot 500 \Rightarrow n' = 550 \text{ Hz}$$



**Problem 44.** A motor car blowing a horn of frequency 124 vibration/sec moves with a velocity 72 km/hr towards a tall wall. The frequency of the reflected sound heard by the driver will be (velocity of sound in air is 330 m/s)

- (a) 109 vibration/sec      (b) 132 vibration/sec      (c) 140 vibration/sec      (d) 248 vibration/sec

**Solution:** (c) In the given condition source and listener are at the same position i.e. (car) for given condition

$$n' = \frac{v + v_{car}}{v - v_{car}} \cdot n = \frac{330 + 20}{330 - 20} \cdot n = 140 \text{ vibration/sec}$$

**Problem 45.** The driver of car travelling with a speed 30 meter/sec. towards a hill sounds a horn of frequency 600 Hz. If the velocity of sound in air is 330 m/s the frequency of reflected sound as heard by the driver is

- (a) 720 Hz                      (b) 555.5 Hz                      (c) 550 Hz                      (d) 500 Hz

**Solution:** (a) This question is same as that of previous one so 
$$n' = \frac{v + v_{car}}{v - v_{car}} \cdot n = 720 \text{ Hz}$$

**Problem 46.** The source of sound s is moving with a velocity 50 m/s towards a stationary observer. The observer measures the frequency of the source as 1000 Hz. What will be the apparent frequency of the source when it is moving away from the observer after crossing him? The velocity of sound in the medium is 350 m/s

- (a) 750 Hz                      (b) 857 Hz                      (c) 1143 Hz                      (d) 1333 Hz

**Solution:** (a) When source is moving towards the stationary listener.

$$n' = \frac{v}{v - v_s} n \Rightarrow 1000 = \frac{350}{350 - 50} \cdot n \Rightarrow n = 857.14$$



When source is moving away from the stationary observer  $n'' = \frac{v}{v + v_s} = \frac{350}{350 + 50} \times 857 = 750 \text{ Hz}$

**Problem 47.** A source and listener are both moving towards each other with speed  $v/10$  where  $v$  is the speed of sound. If the frequency of the note emitted by the source is  $f$ , the frequency heard by the listener would be nearly

- (a)  $1.11 f$                       (b)  $1.22 f$                       (c)  $f$                       (d)  $1.27 f$

**Solution:** (b)  $n' = \left( \frac{v + v_L}{v - v_s} \right) n \Rightarrow n' = \left( \frac{v + \frac{v}{10}}{v - \frac{v}{10}} \right) n \Rightarrow n' = \frac{11}{9} f = 1.22 f.$

**Problem 48.** A man is watching two trains, one leaving and the other coming in with equal speed of  $4 \text{ m/s}$ . If they sound their whistles, each of frequency  $240 \text{ Hz}$ , the number of beats heard by the man (velocity of sound in air =  $320 \text{ m/s}$ ) will be equal to

- (a) 6                      (b) 3                      (c) 0                      (d) 12

**Solution:** (a) App. Frequency due to train which is coming in  $n_1 = \frac{v}{v - v_s} . n$

App. Frequency due to train which is leaving  $n_2 = \frac{v}{v + v_s} . n$

So number of beats  $n_1 - n_2 = \left( \frac{1}{316} - \frac{1}{324} \right) 320 \times 240 \Rightarrow n_1 - n_2 = 6$

**Problem 49.** At what speed should a source of sound move so that observer finds the apparent frequency equal to half of the original frequency

- (a)  $v/2$                       (b)  $2v$                       (c)  $v/4$                       (d)  $v$

**Solution:** (d)  $n' = \frac{v}{v + v_s} . n \Rightarrow \frac{n}{2} = \frac{v}{v + v_s} . n \Rightarrow v_s = v$



# Practice Problems

## Problems based on Progressive waves

### ► Basic level

- In a progressive wave, the distance between two consecutive crests is
  - $\lambda/2$
  - $\lambda$
  - $3\lambda/2$
  - $2\lambda$
- The equation of a wave is  $y = 3 \cos f(50t - x)$ . The wavelength of the wave is
  - 3 units
  - 2 units
  - 50 units
  - 47 units
- If the wave equation  $y = 0.08 \sin \frac{2\pi}{\lambda}(200t - x)$  then the velocity of the wave will be
  - $400\sqrt{2}$
  - $200\sqrt{2}$
  - 400
  - 200
- A wave of frequency 500 Hz has velocity 360 m/sec. The distance between two nearest points  $60^\circ$  out of phase, is
  - 0.6 cm
  - 12 cm
  - 60 cm
  - 120 cm
- The equation of a transverse wave is given by  $y = 10 \sin f(0.01x - 2t)$  where  $x$  and  $y$  are in cm and  $t$  is in second. Its frequency is
  - 10 sec<sup>-1</sup>
  - 2 sec<sup>-1</sup>
  - 1 sec<sup>-1</sup>
  - 0.01 sec<sup>-1</sup>
- If the frequency of a wave is 360 s<sup>-1</sup>, the distance between two nearest compression & rarefaction is 1m. Then the velocity of wave is
  - 720 m/s
  - 180 m/s
  - 360 m/s
  - 90 m/s
- It takes 2.0 seconds for a sound wave to travel between two fixed points when the day temperature is 10° C. If the temperature rise to 30° C the sound wave travels between the same fixed points in
  - 1.9 sec
  - 2.0 sec
  - 2.1 sec
  - 2.2 sec
- The equation of a wave is given as  $y = N 0.07 \sin(12\pi x + 3000\pi t)$ . Where  $x$  is in metre and  $t$  in sec, then the correct statement is
  - $\lambda = 1/6m, v = 250 m/s$
  - $\lambda = 0.07m, v = 300 m/s$
  - $\lambda = 1500, v = 200 m/s$
  - None of these

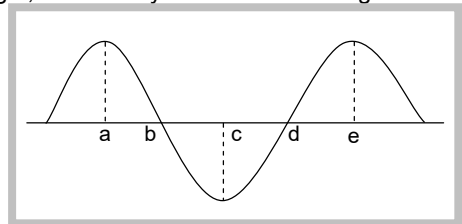
## 38 Wave Motion

9. The equation of the propagating wave is  $y = 25 \sin(20t + 5x)$ , where  $y$  is displacement. Which of the following statement is not true
- (a) The amplitude of the wave is 25 units  
 (b) The wave is propagating in positive  $x$ -direction  
 (c) The velocity of the wave is 4 units  
 (d) The maximum velocity of the particles is 500 units
10. In a plane progressive wave given by  $y = 25 \cos(2ft - fx)$ , the amplitude and frequency are respectively
- (a) 25, 100  
 (b) 25, 1  
 (c) 25, 2  
 (d)  $50f, 2$
11. If  $v_m$  is the velocity of sound in moist air,  $v_d$  is the velocity of sound in dry air, under identical conditions of pressure and temperature
- (a)  $v_m > v_d$   
 (b)  $v_m < v_d$   
 (c)  $v_m = v_d$   
 (d)  $v_m v_d = 1$
12. The displacement  $y$  of a wave travelling in the  $x$ -direction is given by  $y = 10^{-4} \sin 600t - 2x < \frac{f}{3}$  metres, where  $x$  is expressed in metres and  $t$  in seconds. The speed of the wave-motion, in  $ms^{-1}$ , is
- (a) 200  
 (b) 300  
 (c) 600  
 (d) 1200
13. The equation  $y = A \cos^2\left(2fnt - 2f\frac{x}{\lambda}\right)$  represents a wave with
- (a) Amplitude  $A/2$ , frequency  $2n$  and wavelength  $\lambda/2$   
 (b) Amplitude  $A/2$ , frequency  $2n$  and wavelength  $\lambda$   
 (c) Amplitude  $A$ , frequency  $2n$  and wavelength  $\lambda/2$   
 (d) Amplitude  $A$ , frequency  $n$  and wavelength  $\lambda$
14.  $v_1$  and  $v_2$  are the velocities of sound at the same temperature in two monoatomic gases of densities  $\rho_1$  and  $\rho_2$  respectively. If  $\rho_1 / \rho_2 = \frac{1}{4}$  then the ratio of velocities  $v_1$  and  $v_2$  will be
- (a) 1 : 2  
 (b) 4 : 1  
 (c) 2 : 1  
 (d) 1 : 4
15. The temperature at which the speed of sound in air becomes double of its value at  $0^\circ C$  is
- (a)  $273^\circ K$   
 (b)  $546^\circ K$   
 (c)  $1092^\circ K$   
 (d)  $0^\circ K$
16. A wave travelling in positive X-direction with  $A = 0.2m$  has a velocity of  $360 m/sec$ . if  $\lambda = 60m$ , then correct expression for the wave is
- (a)  $y = 0.2 \sin\left[2f\left(6t + \frac{x}{60}\right)\right]$   
 (b)  $y = 0.2 \sin\left[f\left(6t + \frac{x}{60}\right)\right]$   
 (c)  $y = 0.2 \sin\left[2f\left(6t - \frac{x}{60}\right)\right]$   
 (d)  $y = 0.2 \sin\left[f\left(6t - \frac{x}{60}\right)\right]$
17. The equation for spherical progressive wave is
- (a)  $y = N a \sin(\tilde{S}t > kx)$   
 (b)  $y = N \frac{a}{\sqrt{r}} \sin(\tilde{S}t > kx)$   
 (c)  $y = N \frac{a}{2} \sin(\tilde{S}t > kx)$   
 (d)  $y = N \frac{a}{r} \sin(\tilde{S}t > kx)$
18. A stone is dropped into a lake from a tower 500 metre high. The sound of the splash will be heard by the man approximately after

- (a) 11.5 sec                      (b) 21 sec                      (c) 10 sec                      (d) 14 sec
19. The equation of a plane progressive wave is given by  $y = 0.25 \sin(100t + 0.25x)$ . The frequency of this wave would be
- (a)  $\frac{50}{f}$  Hz                      (b)  $\frac{100}{f}$  Hz                      (c) 100 Hz                      (d) 50 Hz
20. The equation of a sound wave is  
 $y = 0.0015 \sin(62.4x + 316t)$   
 The wavelength of this wave is
- (a) 0.2 unit                      (b) 0.1 unit                      (c) 0.3 unit                      (d) Cannot be calculated
21. The equation of a travelling wave is  
 $y = 60 \cos(1800t - 6x)$   
 where  $y$  is in microns,  $t$  in seconds and  $x$  in meters. The ratio of maximum particle velocity to velocity of wave propagation is
- (a)  $3.6 \times 10^{-11}$                       (b)  $3.6 \times 10^{-6}$                       (c)  $3.6 \times 10^{-4}$                       (d) 3.6
22. The wave equation is  $y = 0.30 \sin(314t - 1.57x)$  where  $t$ ,  $x$  and  $y$  are in second, meter and centimeter respectively. The speed of the wave is
- (a) 100 m/s                      (b) 200 m/s                      (c) 300 m/s                      (d) 400 m/s
23. Transverse waves can propagate
- (a) Both in a gas and a metal                      (b) In a gas but not in a metal  
 (c) Not in a gas but in a metal                      (d) Neither in a gas nor in a metal
24. The sound carried by air from a sitar to a listener is a wave of the following type
- (a) Longitudinal stationary    (b) Transverse progressive    (c) Transverse stationary    (d) Longitudinal progressive
25. A tuning fork produces wave in medium. If the temperature of the medium changes then which of following will change
- (a) Time period                      (b) Wavelength                      (c) Frequency                      (d) Amplitude
26. The equation of a longitudinal wave is represented as  $y = 20 \cos f(50t + x)$ . Its wavelength is
- (a) 5 m                      (b) 2 m                      (c) 50 m                      (d) 20 m
27. The rope shown at an instant is carrying a wave travelling towards right, created by a source vibrating at a frequency  $n$ . Consider the following statements

I. The speed of the wave is  $4n \times ab$

II. The medium at  $a$  will be in the same phase as  $d$  after  $\frac{4}{3n}$  s



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III. The phase difference between  $b$  and  $e$  is  $\frac{3f}{2}$

Which of these statements are correct

- (a) I, II and III                      (b) II only                      (c) I and III                      (d) III only

28. To increase the frequency from 100 Hz to 400 Hz the tension in the string has to be changed by

- (a) 4 times                      (b) 16 times                      (c) 20 times                      (d) None of these

29. Velocity of sound in air

- I. Increases with temperature                      II. Decreases with temperature  
 III. Increase with pressure                      IV. Is independent of pressure  
 V. Is independent of temperature

Choose the correct answer.

- (a) Only I and II are true                      (b) Only I and III are true                      (c) Only II and III are true                      (d) Only I and IV are true

30. The speed of a wave in a medium is 760 m/s. If 3600 waves are passing through a point, in the medium in 2 minutes, then its wavelength is

- (a) 13.8 m                      (b) 25.3 m                      (c) 41.5 m                      (d) 57.2 m

31. A string of 7 m length has a mass of 0.035 kg. If tension in the string is 60.5 N, then speed of a wave on the string is

- (a) 77 m/s                      (b) 102 m/s                      (c) 110 m/s                      (d) 165 m/s

32. The relation between phase difference and path difference is

- (a)  $\Delta w = \frac{2f}{\lambda} \Delta x$                       (b)  $\Delta w = 2f \lambda \Delta x$                       (c)  $\Delta w = \frac{2f \lambda}{\Delta x}$                       (d)  $\Delta w = \lambda \frac{2 \Delta x}{\lambda}$

33. The frequency of a rod is 200 Hz. If the velocity of sound in air is 340 ms<sup>-1</sup>, the wavelength of the sound produced is

- (a) 1.7 cm                      (b) 6.8 cm                      (c) 1.7 m                      (d) 6.8 m

34. If the pressure amplitude in a sound wave is tripled, then the intensity of sound is increased by a factor of

- (a) 9                      (b) 3                      (c) 6                      (d)  $\sqrt{3}$

35. Two monoatomic ideal gases 1 and 2 of molecular masses  $m_1$  and  $m_2$  respectively are enclosed in separate containers kept at the same temperature. The ratio of the speed of sound in gas 1 to that in gas 2 is given by

- (a)  $\sqrt{\frac{m_1}{m_2}}$                       (b)  $\sqrt{\frac{m_2}{m_1}}$                       (c)  $\frac{m_1}{m_2}$                       (d)  $\frac{m_2}{m_1}$

36. A man is standing between two parallel cliffs and fires a gun. If he hears first and second echoes after 1.5 s and 3.5 s respectively, the distance between the cliffs is (Velocity of sound in air = 340 ms<sup>-1</sup>)

- (a) 1190 m                      (b) 850 m                      (c) 595 m                      (d) 510 m

37. When the temperature of an ideal gas is increased by 600 K, the velocity of sound in the gas becomes  $\sqrt{3}$  times the initial velocity in it. The initial temperature of the gas is



- (a)  $> 73^\circ C$                       (b)  $27^\circ C$                       (c)  $127^\circ C$                       (d)  $327^\circ C$

38. The frequency of a sound wave is  $n$  and its velocity is  $v$ . If the frequency is increased to  $4n$ , the velocity of the wave will be

- (a)  $v$                       (b)  $2v$                       (c)  $4v$                       (d)  $v/4$

39. In a transverse progressive wave of amplitude  $A$ , the maximum particle velocity is four times of its wave velocity. The wavelength of the wave is

- (a)  $\frac{fA}{4}$                       (b)  $\frac{fA}{2}$                       (c)  $fA$                       (d)  $2fA$

40. A man fires a bullet standing between two cliffs. First echo is heard after 3 seconds and second echo is heard after 5 seconds. If the velocity of sound is  $330 \text{ m/s}$ , then the distance between the cliffs is

- (a)  $1650 \text{ m}$                       (b)  $1320 \text{ m}$                       (c)  $990 \text{ m}$                       (d)  $660 \text{ m}$

41. A string on a musical instrument is  $50 \text{ cm}$  long and its fundamental frequency is  $270 \text{ Hz}$ . If the desired frequency of  $1000 \text{ Hz}$  is to be produced, the required length of the string is

- (a)  $13.5 \text{ cm}$                       (b)  $2.7 \text{ cm}$                       (c)  $5.4 \text{ cm}$                       (d)  $10.3 \text{ cm}$

42. Consider the following statements.

**Assertion (A)** : Like sound, light can not propagate in vacuum.

**Reason (R)** : Sound is a square wave. It propagates in a medium by a virtue of damping oscillation

Of these statements

- (a) Both  $A$  and  $R$  are true and the  $R$  is a correct explanation of the  $A$   
 (b) Both  $A$  and  $R$  are true but the  $R$  is not a correct explanation of the  $A$   
 (c)  $A$  is true but the  $R$  is false  
 (d) Both  $A$  and  $R$  are false  
 (e)  $A$  is false but the  $R$  is true

43. Sound velocity is maximum in

- (a)  $H_2$                       (b)  $N_2$                       (c)  $He$                       (d)  $O_2$

44. The minimum distance of reflector surface from the source for listening the echo of sound is

- (a)  $28 \text{ m}$                       (b)  $18 \text{ m}$                       (c)  $19 \text{ m}$                       (d)  $16.5 \text{ m}$

45. A transverse wave is described by the equation  $Y = Y_0 \sin 2f \left( ft - \frac{x}{\lambda} \right)$ . The maximum particle velocity is four times the wave velocity if

- (a)  $\lambda = \frac{fY_0}{4}$                       (b)  $\lambda = \frac{fY_0}{2}$                       (c)  $\lambda = fY_0$                       (d)  $\lambda = 2fY_0$

46. The equation of a wave travelling in a string can be written as  $y = 3 \cos f(100t - x)$ . Its wavelength is

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- (a) 100 cm                      (b) 2 cm                      (c) 5 cm                      (d) None of the above

47. Which of the property makes difference between progressive and stationary waves

- (a) Amplitude                      (b) Frequency                      (c) Propagation of energy                      (d) Phase of the wave

48. Which of the following equation does not represent the progressive wave

- (a)  $y = A \sin \left[ \left( t - \frac{x}{v} \right) \right]$                       (b)  $y = A \sin 2f \left( \frac{t}{T} + \frac{x}{\lambda} \right)$                       (c)  $y = A \sin \frac{2f}{\lambda} (vt - x)$                       (d)  $y = A \sin 2f \left( \frac{t}{T} - \frac{x}{v} \right)$

### *Problems based on Superposition of waves*

49. In an open organ pipe.....wave is present

- (a) Transverse standing wave                      (b) Longitudinal standing wave  
(c) Longitudinal moving wave                      (d) Transverse moving wave

50. Two waves are propagating to the point  $P$  along a straight line produced by two sources  $A$  and  $B$  of simple harmonic and of equal frequency. The amplitude of every wave at  $P$  is ' $a$ ' and the phase of  $A$  is ahead by  $\frac{f}{3}$  than that of  $B$  and the distance  $AP$  is greater than  $BP$  by 50 cm. Then the resultant amplitude at the point  $P$  will be, if the wavelength is 1 meter

- (a)  $2a$                       (b)  $a\sqrt{3}$                       (c)  $a\sqrt{2}$                       (d)  $a$

51. Two tuning forks have frequencies 450 Hz and 454 Hz respectively. On sounding these forks together, the time interval between successive maximum intensities will be

- (a) 1/4 sec                      (b) 1/2 sec                      (c) 1 sec                      (d) 2 sec

52. Two waves of lengths 50 cm and 51 cm produced 12 beats per second. The velocity of sound is

- (a) 306 m/s                      (b) 331 m/s                      (c) 340 m/s                      (d) 360 m/s

53. In stationary longitudinal waves, nodes are points of

- (a) Minimum pressure                      (b) Maximum pressure  
(c) Minimum pressure variation                      (d) Maximum pressure variation

54. A cylindrical tube, open at both ends, has a fundamental frequency  $f_0$  in air. The tube is dipped vertically into water such that half of its length is inside water. The fundamental frequency of the air column now is

- (a)  $3f_0/4$                       (b)  $f_0$                       (c)  $f_0/2$                       (d)  $2f_0$

55. Equation of motion in the same direction is given by  $y_1 = A \sin(\tilde{S}t - kx)$ ,  $y_2 = A \sin(\tilde{S}t - kx - \pi)$ . The amplitude of the medium particle will be

- (a)  $2A \cos \frac{\pi}{2}$                       (b)  $2A \cos \pi$                       (c)  $f, 1.2\}$                       (d)  $1.2f, 1.2\}$

56. A closed organ pipe and an open organ pipe are tuned to the same fundamental frequency. What is the ratio of lengths

- (a) 1 : 2                      (b) 2 : 1                      (c) 2 : 3                      (d) 4 : 3
57. An open pipe resonates with a tuning fork of frequency 500 Hz. It is observed that two successive nodes are formed at distances 16 and 46 cm from the open end. The speed of sound in air in the pipe is
- (a) 230 m/s                      (b) 300 m/s                      (c) 320 m/s                      (d) 360 m/s
58. In the experiment for the determination of the speed of sound in air using the resonance column method, the length of the air column that resonates in the fundamental mode, with a tuning fork is 0.1 m. When this length is changed to 0.35 m, the same tuning fork resonates with the first overtone. Calculate the end correction
- (a) 0.012m                      (b) 0.025m                      (c) 0.05m                      (d) 0.024m
59. Two sound sources when sounded simultaneously produce four beats in 0.25 second. The difference in their frequencies must be
- (a) 4                              (b) 8                              (c) 16                              (d) 1
60. At nodes in stationary waves
- (a) Change in pressure and density are maximum                      (b) Change in pressure and density are minimum
- (c) Strain is zero                      (d) Energy is minimum
61. Find the fundamental frequency of a closed pipe, if the length of the air column is 42 m. (speed of sound in air = 332 m/sec)
- (a) 2 Hz                              (b) 4 Hz                              (c) 7 Hz                              (d) 9 Hz
62. If  $v$  is the speed of sound in air then the shortest length of the closed pipe which resonates to a frequency  $n$
- (a)  $\frac{v}{4n}$                               (b)  $\frac{v}{2n}$                               (c)  $\frac{2n}{v}$                               (d)  $\frac{4n}{v}$
63. Two uniform strings  $A$  and  $B$  made of steel are made to vibrate under the same tension. If the first overtone of  $A$  is equal to the second overtone of  $B$  and if the radius of  $A$  is twice that of  $B$ , the ratio of the lengths of the strings is
- (a) 1 : 2                              (b) 1 : 3                              (c) 1 : 4                              (d) 1 : 6
64. If the length of a stretched string is shortened by 40% and the tension is increased by 44%, then the ratio of the final and initial fundamental frequencies is
- (a) 2 : 1                              (b) 3 : 2                              (c) 3 : 4                              (d) 1 : 3
65. Two wires are fixed in a sonometer. Their tensions are in the ratio 8 : 1. The lengths are in the ratio 36 : 35. The diameters are in the ratio 4 : 1. Densities of the materials are in the ratio 1 : 2. If the higher frequency in the setting is 360 Hz. The beat frequency when the two wires are sounded together is
- (a) 5                              (b) 8                              (c) 6                              (d) 10
66. A metal wire of linear mass density of 9.8 g/m is stretched with a tension of 10 kg weight between two rigid supports 1 metre apart. The wire passes at its middle point between the poles of a permanent magnet, and it vibrates in resonance when carrying an alternating current of frequency  $n$ . The frequency  $n$  of the alternating source is
- (a) 25 Hz                              (b) 50 Hz                              (c) 100 Hz                              (d) 200 Hz

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67. A tuning fork of known frequency  $256\text{ Hz}$  makes 5 beats per second with the vibrating string of a piano. The beat frequency decreases to 2 beats per second when the tension in the piano string is slightly increased. The frequency of the piano string before increasing the tension was
- (a)  $256 + 5\text{ Hz}$                       (b)  $256 + 2\text{ Hz}$                       (c)  $256 - 2\text{ Hz}$                       (d)  $256 - 5\text{ Hz}$
68. The frequency of fundamental tone in an open organ pipe of length  $0.48\text{ m}$  is  $320\text{ Hz}$ . Speed of sound is  $320\text{ m/sec}$ . Frequency of fundamental tone in closed organ pipe will be
- (a)  $153.8\text{ Hz}$                       (b)  $160.0\text{ Hz}$                       (c)  $320.0\text{ Hz}$                       (d)  $143.2\text{ Hz}$
69. A sonometer wire resonates with a given tuning fork forming standing waves with five antinodes between the two bridges when a mass of  $9\text{ kg}$  is suspended from the wire. When this mass is replaced by a mass  $M$ , the wire resonates with the same tuning fork forming three antinodes for the same positions of the bridges. The value of  $M$  is
- (a)  $25\text{ kg}$                       (b)  $5\text{ kg}$                       (c)  $12.5\text{ kg}$                       (d)  $1/25\text{ kg}$
70. The tension of a stretched string is increased by  $69\%$ . In order to keep its frequency of vibration constant, its length must be increased by
- (a)  $20\%$                       (b)  $30\%$                       (c)  $\sqrt{69}\%$                       (d)  $69\%$
71. A tuning fork arrangement (pair) produces  $4\text{ beats/sec}$  with one fork of frequency  $288\text{ cps}$ . A little wax is placed on the unknown fork and it then produces  $2\text{ beats/sec}$ . The frequency of the unknown fork is
- (a)  $286\text{ cps}$                       (b)  $292\text{ cps}$                       (c)  $294\text{ cps}$                       (d)  $288\text{ cps}$
72. Two wires are in unison. If the tension in one of the wires is increased by  $2\%$ , 5 beats are produced per second. The initial frequency of each wire is
- (a)  $200\text{ Hz}$                       (b)  $400\text{ Hz}$                       (c)  $500\text{ Hz}$                       (d)  $1000\text{ Hz}$
73. Two closed organ pipes, when sounded simultaneously gave 4 beats per sec. If longer pipe has a length of  $1\text{ m}$ . Then length of shorter pipe will be, ( $v = 300\text{ m/s}$ )
- (a)  $185.5\text{ cm}$                       (b)  $94.9\text{ cm}$                       (c)  $90\text{ cm}$                       (d)  $80\text{ cm}$
74. A source of sound placed at the open end of a resonance column sends an acoustic wave of pressure amplitude  $P_0$  inside the tube. If the atmospheric pressure is  $P_A$ , then the maximum and minimum pressure at the closed end of the tube will be
- (a)  $(P_A + P_0), (P_A - P_0)$                       (b)  $(P_A + 2P_0), (P_A - 2P_0)$                       (c)  $P_A, P_A$                       (d)  $\left(P_A + \frac{1}{2}P_0\right), \left(P_A - \frac{1}{2}P_0\right)$
75. Ten tuning forks are arranged in increasing order of frequency in such a way that any two nearest tuning forks produce 4 beats/sec. The highest frequency is twice of the lowest. Possible highest and the lowest frequencies are
- (a) 80 and 40                      (b) 100 and 50                      (c) 44 and 22                      (d) 72 and 36
76. If two waves of same frequency and same amplitude respectively, on superimposition produced a resultant disturbance of the same amplitude, the waves differ in phase by
- (a)  $f$                       (b)  $2f/3$                       (c)  $f/2$                       (d) Zero
77. In stationary waves all particles between two nodes pass through the mean position

- (a) At different times with different velocities                      (b) At different times with the same velocity  
 (c) At the same time with equal velocity                              (d) At the same time with different velocities
78. For production of beats, the two sources must have  
 (a) Different frequencies and same amplitude                      (b) Different frequencies  
 (c) Different frequencies, same amplitude and same phase      (d) Different frequencies and same phase
79. Sixteen tuning forks are arranged in order of increasing frequencies. Adjacent successive forks, when sounded together, give 8 beats per second. If the frequency of the last tuning fork is twice that of the first fork, the frequency of the last fork is  
 (a) 256 Hz                      (b) 240 Hz                      (c) 128 Hz                      (d) 120 Hz
80. It is possible to hear beats from the two vibrating sources of frequency  
 (a) 100 Hz and 150 Hz      (b) 20 Hz and 25 Hz                      (c) 400 Hz and 500 Hz      (d) 1000 Hz and 1500 Hz
81. The ends of a stretched wire of length  $L$  are fixed at  $x = 0$  and  $x = L$ . In one experiment, the displacement of the wire is  $y_1 = A \sin(\pi x / L) \sin \pi t$  and energy is  $E_1$ , and in another experiment its displacement is  $y_2 = A \sin(2\pi x / L) \sin 2\pi t$  and energy is  $E_2$ . Then  
 (a)  $E_2 = E_1$                       (b)  $E_2 = 2E_1$                       (c)  $E_2 = 4E_1$                       (d)  $E_2 = 16E_1$
82. Two pulses in a stretched string whose centres are initially 8 cm apart are moving towards each other as shown in the figure. The speed of each pulse is 2 cm/s. After 2 seconds, the total energy of the pulses will be  
 (a) Zero  
 (b) Purely kinetic  
 (c) Purely potential  
 (d) Partly kinetic and partly potential
- The diagram shows a horizontal string with two pulses. The left pulse is a crest (upward bump) moving to the right, indicated by a right-pointing arrow above it. The right pulse is a trough (downward bump) moving to the left, indicated by a left-pointing arrow above it. A double-headed arrow below the string indicates the initial distance between the centers of the two pulses is 8 cm.
83. In order to double the frequency of the fundamental note emitted by a stretched string, the length is reduced to  $\frac{3}{4}$  of the original length and the tension is changed. The factor by which the tension is to be changed, is  
 (a)  $\frac{3}{8}$                       (b)  $\frac{2}{3}$                       (c)  $\frac{8}{9}$                       (d)  $\frac{9}{4}$
84. Two sound waves of wavelengths 5 m and 6 m formed 30 beats in 3 seconds. The velocity of sound is  
 (a) 300  $ms^{-1}$                       (b) 310  $ms^{-1}$                       (c) 320  $ms^{-1}$                       (d) 330  $ms^{-1}$
85. If the length of a closed organ pipe is 1 m and velocity of sound is 330 m/s, then the frequency for the second note is  
 (a)  $4 \times \frac{330}{4}$  Hz                      (b)  $3 \times \frac{330}{4}$  Hz                      (c)  $2 \times \frac{330}{4}$  Hz                      (d)  $2 \times \frac{4}{330}$  Hz

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86. The fundamental note produced by a closed organ pipe is of frequency  $f$ . The fundamental note produced by an open organ pipe of same length will be of frequency
- (a)  $\frac{f}{2}$                       (b)  $f$                       (c)  $2f$                       (d)  $4f$
87. Two open organ pipes give 4 beats/sec, when sounded together in their fundamental notes. If the length of the pipes are 100 cm and 102.5 cm respectively, then the velocity of sound is
- (a) 160 m/s                      (b) 240 m/s                      (c) 328 m/s                      (d) 496 m/s
88. A second harmonic has to be generated in a string of length  $l$  stretched between two rigid supports. The point where the string has to be plucked and touched are
- (a) Plucked at  $\frac{l}{4}$  and touch at  $\frac{l}{2}$                       (b) Plucked at  $\frac{l}{4}$  and touch at  $\frac{3l}{4}$   
(c) Plucked at  $\frac{l}{2}$  and touched at  $\frac{l}{4}$                       (d) Plucked at  $\frac{l}{2}$  and touched at  $\frac{3l}{4}$
89. If the velocity of sound in air is 336 m/s. The maximum length of a closed pipe that would produce a just audible sound will be
- (a) 3.2 cm                      (b) 4.2 m                      (c) 4.2 cm                      (d) 3.2 m
90. A resonance air column of length 20 cm resonates with a tuning fork of frequency 250 Hz. The speed of the air is
- (a) 300 m/s                      (b) 200 m/s                      (c) 150 m/s                      (d) 75 m/s
91. Two waves are approaching each other with a velocity of 16 m/s and frequency  $n$ . The distance between two consecutive nodes is
- (a)  $\frac{16}{n}$                       (b)  $\frac{8}{n}$                       (c)  $\frac{n}{16}$                       (d)  $\frac{n}{8}$
92. An organ pipe  $P_1$  closed at one end vibrating in its first overtone and another pipe  $P_2$  open at both ends vibrating in its third overtone are in resonance with a given tuning fork. The ratio of lengths of  $P_1$  and  $P_2$  is
- (a) 1 : 2                      (b) 1 : 3                      (c) 3 : 8                      (d) 3 : 4
93. 16 tuning forks are arranged in the order of increasing frequencies. Any two successive forks give 8 beats per sec when sounded together. If the frequency of the last fork is twice the first, then the frequency of the first fork is
- (a) 120                      (b) 160                      (c) 180                      (d) 220
94. Two waves  $y = 0.25 \sin 316t$  and  $y = 0.25 \sin 310t$  are travelling in same direction. The number of beats produced per second will be
- (a) 6                      (b) 3                      (c)  $3/f$                       (d)  $3f$
95. If the temperature increases, then what happens to the frequency of the sound produced by the organ pipe

- (a) Increases                      (b) Decreases                      (c) Unchanged                      (d) Not definite
96. Standing waves are produced in a 10 m long stretched string. If the string vibrates in 5 segments and the wave velocity is 20 m/s, the frequency is  
 (a) 2 Hz                      (b) 4 Hz                      (c) 5 Hz                      (d) 10 Hz
97. An unknown frequency  $x$  produces 8 beats per seconds with a frequency of 250 Hz and 12 beats with 270 Hz source, then  $x$  is  
 (a) 258 Hz                      (b) 242 Hz                      (c) 262 Hz                      (d) 282 Hz
98.  $y = a \cos(kx + \tilde{S}t)$  superimposes on another wave giving a stationary wave having node at  $x = 0$ . What is the equation of the other wave  
 (a)  $-a \cos(kx + \tilde{S}t)$                       (b)  $a \cos(kx - \tilde{S}t)$                       (c)  $-a \cos(kx - \tilde{S}t)$                       (d)  $-a \sin(kx + \tilde{S}t)$
99. Two sound waves of slightly different frequencies propagating in the same direction produce beats due to  
 (a) Interference                      (b) Diffraction                      (c) Polarization                      (d) Refraction
100. On sounding tuning fork  $A$  with another tuning fork  $B$  of frequency 384 Hz, 6 beats are produced per second. After loading the prongs of  $A$  with some wax and then sounding it again with  $B$ , 4 beats are produced per second. What is the frequency of the tuning fork  $A$   
 (a) 388 Hz                      (b) 380 Hz                      (c) 378 Hz                      (d) 390 Hz
101. Four wires of identical length, diameters and of the same material are stretched on a sonometre wire. If the ratio of their tensions is 1 : 4 : 9 : 16 then the ratio of their fundamental frequencies are  
 (a) 16 : 9 : 4 : 1                      (b) 4 : 3 : 2 : 1                      (c) 1 : 4 : 2 : 16                      (d) 1 : 2 : 3 : 4
102. If you set up the ninth harmonic on a string fixed at both ends, what is its frequency compared to the seventh harmonic  
 (a) Higher                      (b) Lower                      (c) Equal                      (d) None of the above
103. The frequency of a stretched uniform wire under tension is in resonance with the fundamental frequency of a closed tube. If the tension in the wire is increased by 8 N, it is in resonance with the first overtone of the closed tube. The initial tension in the wire is  
 (a) 1 N                      (b) 4 N                      (c) 8 N                      (d) 16 N
104. Two waves  $y_1 = A_1 \sin(\tilde{S}t - s_1)$  and  $y_2 = A_2 \sin(\tilde{S}t - s_2)$  Superimpose to form a resultant wave whose amplitude is  
 (a)  $\sqrt{A_1^2 + A_2^2 + 2A_1A_2 \cos(s_1 - s_2)}$                       (b)  $\sqrt{A_1^2 + A_2^2 + 2A_1A_2 \sin(s_1 - s_2)}$   
 (c)  $A_1 + A_2$                       (d)  $|A_1 + A_2|$
105. In stationary wave  
 (a) Strain is maximum at nodes                      (b) Strain is maximum at antinodes

- (c) Strain is minimum at nodes (d) Amplitude is zero at all the points
106. A wave of frequency  $100\text{ Hz}$  is sent along a string towards a fixed end. When this wave travels back after reflection, a node is formed at a distance of  $10\text{ cm}$  from the fixed end of the string. The speed of incident (and reflected) wave are
- (a)  $40\text{ m/s}$  (b)  $20\text{ m/s}$  (c)  $10\text{ m/s}$  (d)  $5\text{ m/s}$
107. The stationary wave  $y = 2a \sin kx \cos \check{S}t$  in a closed organ pipe is the result of the superposition of  $y = a \sin(\check{S}t + kx)$  and
- (a)  $y = -a \cos(\check{S}t + kx)$  (b)  $y = -a \sin(\check{S}t + kx)$  (c)  $y = a \sin(\check{S}t - kx)$  (d)  $y = a \cos(\check{S}t + kx)$
108. Out of the given four waves
- $y = a \sin(kx + \check{S}t)$  .....(1)  $y = a \sin(\check{S}t - kx)$  .....(2)
- $y = a \cos(kx + \check{S}t)$  .....(3)  $y = a \cos(\check{S}t - kx)$  .....(4)
- Emitted by four different sources  $S_1, S_2, S_3$  and  $S_4$  respectively, interference phenomena would be observed in space under appropriate conditions when
- (a) Source  $S_1$  emits wave (1) and  $S_4$  emits wave (4)
- (b) Source  $S_2$  emits wave (2) and  $S_4$  emits wave (4)
- (c) Source  $S_1$  emits wave (1) and  $S_2$  emits wave (3)
- (d) Interference phenomenon cannot be observed by the combination of any of the above waves
109. The phase difference between the two particles situated on both the sides of a node is
- (a)  $0^\circ$  (b)  $90^\circ$  (c)  $180^\circ$  (d)  $360^\circ$
110. In large room, a person receives direct sound waves from a source  $120\text{ meters}$  away from him. He also receive waves from the same source which reach him, being reflected from the  $25\text{ meter}$  high ceiling at the point halfway between them. The two waves interfere constructively for wavelength of
- (a)  $20, 20/3, 20/5\text{ etc.}$  (b)  $10, 5, 2.5\text{ etc.}$  (c)  $10, 20, 30\text{ etc}$  (d)  $15, 25, 35\text{ etc}$

### *Problems based on Doppler's effect*

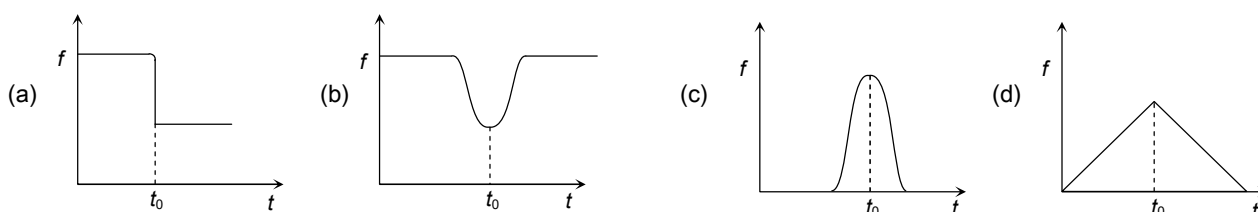
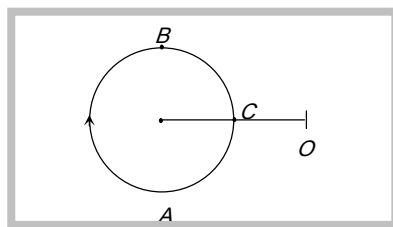
111. Doppler effect is independent of
- (a) Distance between source and listener (b) Velocity of source
- (c) Velocity of listener (d) None of these
112. A source and an observer approach each other with same velocity  $50\text{ m/s}$ . If the apparent frequency is  $435\text{ s}^{-1}$ , then the real frequency is
- (a)  $320\text{ s}^{-1}$  (b)  $360\text{ s}^{-1}$  (c)  $390\text{ s}^{-1}$  (d)  $420\text{ s}^{-1}$
113. A source emits a sound of frequency of  $400\text{ Hz}$ , but the listener hears it to be  $390\text{ Hz}$ . Then
- (a) The listener is moving towards the source (b) The source is moving towards the listener



- (c) The listener is moving away from the source                      (d) The listener has a defective ear
114. A source and an observer are moving towards each other with a speed equal to  $\frac{v}{2}$  where  $v$  is the speed of sound. The source is emitting sound of frequency  $n$ . The frequency heard by the observer will be
- (a) Zero                      (b)  $n$                       (c)  $\frac{n}{3}$                       (d)  $3n$
115. A police car moving at  $22 \text{ m/s}$ , chases a motorcyclist. The police man sounds his horn at  $176 \text{ Hz}$ , while both of them move towards a stationary siren of frequency  $165 \text{ Hz}$ . Calculate the speed of the motorcycle, if it is given that he does not observe any beats
- Police Car                      Motorcycle                      Stationary siren  
                                             
 $\xrightarrow{22 \text{ m/s}}$                        $\xrightarrow{v}$                        $(165 \text{ Hz})$   
 $(176 \text{ Hz})$
- (a)  $33 \text{ m/s}$   
 (b)  $22 \text{ m/s}$   
 (c) Zero  
 (d)  $11 \text{ m/s}$
116. An observer moves towards a stationary source of sound with a speed  $\frac{1}{5}$ th of the speed of sound. The wavelength and frequency of the source emitted are  $\lambda$  and  $f$  respectively. The apparent frequency and wavelength recorded by the observer are respectively
- (a)  $1.2 f, \lambda$                       (b)  $f, 1.2 \lambda$                       (c)  $0.8 f, 0.8 \lambda$                       (d)  $1.2 f, 1.2 \lambda$
117. When an engine passes near to a stationary observer then its apparent frequencies occurs in the ratio  $5/3$ . If the velocity of engine is
- (a)  $540 \text{ m/s}$                       (b)  $270 \text{ m/s}$                       (c)  $85 \text{ m/s}$                       (d)  $52.5 \text{ m/s}$
118. A siren placed at a railway platform is emitting sound of frequency  $5 \text{ kHz}$ . A passenger sitting in a moving train  $A$  records a frequency of  $5.5 \text{ kHz}$  while the train approaches the siren. During his return journey in a different train  $B$  he records a frequency of  $6.0 \text{ kHz}$  while approaching the same siren. The ratio of the velocity of train  $B$  to that of train  $A$  is
- (a)  $242/252$                       (b)  $2$                       (c)  $5/6$                       (d)  $11/6$
119. A racing car moving towards a cliff, sounds its horn. The driver observes that the sound reflected from the cliff has a pitch one octave higher than the actual sound of the horn. If  $v$  is the velocity of sound, then the velocity of the car is
- (a)  $v/\sqrt{2}$                       (b)  $v/2$                       (c)  $v/3$                       (d)  $v/4$
120. A person carrying a whistle emitting continuously a note of  $272 \text{ Hz}$  is running towards a reflecting surface with a speed of  $18 \text{ km/hour}$ . The speed of sound in air is  $345 \text{ m/s}$ . The number of beats heard by him is
- (a) 4                      (b) 6                      (c) 8                      (d) 3

## 50 Wave Motion

121. A bus is moving with a velocity of  $5 \text{ m/s}$  towards a huge wall. the driver sounds a horn of frequency  $165 \text{ Hz}$ . If the speed of sound in air is  $355 \text{ m/s}$ , the number of beats heard per second by a passenger on the bus will be  
 (a) 6 (b) 5 (c) 3 (d) 4
122. A car sounding a horn of frequency  $1000 \text{ Hz}$  passes an observer. The ratio of frequencies of the horn noted by the observer before and after passing of the car is  $11 : 9$ . If the speed of sound is  $v$ , the speed of the car is  
 (a)  $\frac{1}{10}v$  (b)  $\frac{1}{2}v$  (c)  $\frac{1}{5}v$  (d)  $v$
123. What should be the velocity of a sound source moving towards a stationary observer so that apparent frequency is double the actual frequency (Velocity of sound is  $v$ )  
 (a)  $v$  (b)  $2v$  (c)  $\frac{v}{2}$  (d)  $\frac{v}{4}$
124. Two trains are moving towards each other at speeds of  $20 \text{ m/s}$  and  $15 \text{ m/s}$  relative to the ground. The first train sounds a whistle of frequency  $600 \text{ Hz}$ . the frequency of the whistle heard by a passenger in the second train before the train meets is (the speed of sound in air is  $340 \text{ m/s}$ )  
 (a)  $600 \text{ Hz}$  (b)  $585 \text{ Hz}$  (c)  $645 \text{ Hz}$  (d)  $666 \text{ Hz}$
125. A small source of sound moves on a circle as shown in the figure and an observer is sitting on  $O$ . Let  $n_1, n_2$  and  $n_3$  be the frequencies heard when the source is at  $A, B$  and  $C$  respectively. Then  
 (a)  $n_1 > n_2 > n_3$   
 (b)  $n_2 > n_3 > n_1$   
 (c)  $n_1 = n_2 > n_3$   
 (d)  $n_2 > n_1 > n_3$





138. A boy is walking away from a wall towards an observer at a speed of 1 *meter/second* and blows a whistle whose frequency is 680 *Hz*. The number of beats heard by the observer per second is (Velocity of sound in air = 340 *meters/sec*)
- (a) Zero                      (b) 2                      (c) 8                      (d) 4

### *Miscellaneous problems*

139. If fundamental frequency of closed pipe is 50 *Hz* then frequency of 2<sup>nd</sup> overtone is

(a) 100 *Hz*                      (b) 50 *Hz*                      (c) 250 *Hz*                      (d) 150 *Hz*

140. The phase difference between two waves, represented by

$$y_1 = 10^{-6} \sin[100t + (x/50) + 0.5]m$$

$$y_2 = 10^{-6} \cos[100t + (x/50)]m$$

where  $x$  is expressed in meters and  $t$  is expressed in seconds, is approximately

(a) 1.07 radians                      (b) 2.07 radians                      (c) 0.5 radians                      (d) 1.5 radians

141. In forced oscillation of a particle the amplitude is maximum for a frequency  $\tilde{S}_1$  of the force, while the energy is maximum for a frequency  $\tilde{S}_2$  of the force, then

(a)  $\tilde{S}_1 = \tilde{S}_2$

(b)  $\tilde{S}_1 > \tilde{S}_2$

(c)  $\tilde{S}_1 < \tilde{S}_2$  when damping is small and  $\tilde{S}_1 > \tilde{S}_2$  when damping is large

(d)  $\tilde{S}_1 < \tilde{S}_2$

142. A man  $x$  can hear only upto 10 *kHz* and another man  $y$  upto 20 *kHz*. A note of frequency 500 *Hz* is produced before them from a stretched string. Then

(a) Both will hear sounds of same pitch but different quality

(b) Both will hear sounds of different pitch but same quality

(c) Both will hear sounds of different pitch and different quality

(d) Both will hear sounds of same pitch and same quality

143. A light pointer fixed to one prong of a tuning fork touches a vertical plate. The fork is set vibrating and the plate is allowed to fall freely. If eight oscillations are counted when the plate falls through 10 *cm*, the frequency of the tuning fork is

(a) 360 *Hz*

(b) 280 *Hz*

(c) 560 *Hz*

(d) 56 *Hz*

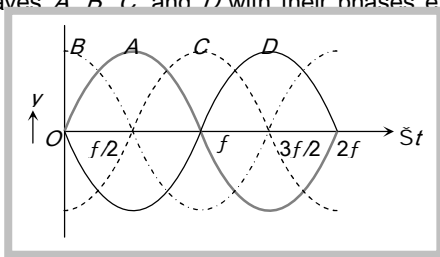
144. Consider the following statements

**Assertion (A)** : The flash of lightening is seen before the sound of thunder is heard.

**Reason (R)** : Speed of sound is greater than speed of light

Of these statements

- (a) Both  $A$  and  $R$  are true and the  $R$  is a correct explanation of the  $A$   
 (b) Both  $A$  and  $R$  are true but the  $R$  is not a correct explanation of the  $A$   
 (c)  $A$  is true but the  $R$  is false  
 (d) Both  $A$  and  $R$  are false  
 (e)  $A$  is false but the  $R$  is true
- 145.** Consider the following
- I. Waves created on the surfaces of a pond water by a vibrating sources.
  - II. Wave created by an oscillating electric field in air.
  - III. Sound waves travelling under water.
- Which of these can be polarized
- (a) I and II                      (b) II only                      (c) II and III                      (d) I, II and III
- 146.** An air column in a pipe, which is closed at one end, will be in resonance with a vibrating body of frequency  $166 \text{ Hz}$ , if the length of the air column is
- (a)  $2.00 \text{ m}$                       (b)  $1.50 \text{ m}$                       (c)  $1.00 \text{ m}$                       (d)  $0.50 \text{ m}$
- 147.** An empty vessel is partially filled with water, then the frequency of vibration of air column in the vessel
- (a) Remains same                      (b) Decreases  
 (c) Increases                      (d) First increases then decreases
- 148.** It is desired to increase the fundamental resonance frequency in a tube which is closed at one end. This can be achieved by
- (a) Replacing the air in the tube by hydrogen gas                      (b) Increasing the length of the tube  
 (c) Decreasing the length of the tube                      (d) Opening the closed end of the tube
- 149.** Quality of a musical note depends on
- (a) Harmonics present                      (b) Amplitude of the wave  
 (c) Fundamental frequency                      (d) Velocity of sound in the medium
- 150.** A wave is reflected from a rigid support. The change in phase on reflection will be
- (a)  $f/4$                       (b)  $f/2$                       (c)  $\pi$                       (d)  $2\pi$
- 151.** The figure shows four progressive waves  $A$ ,  $B$ ,  $C$  and  $D$  with their phases expressed with respect to the wave  $A$ . It can be concluded from the figure that



- (a) The wave  $C$  is ahead by a phase angle of  $f/2$  and the wave  $B$  lags behind by a phase angle of  $f/2$
- (b) The wave  $C$  lags behind by a phase angle of  $f/2$  and the wave  $B$  is ahead by a phase angle of  $f/2$
- (c) The wave  $C$  is ahead by a phase angle of  $f$  and the wave  $B$  lags behind by a phase angle of  $f$
- (d) The wave  $C$  lags behind by a phase angle of  $f$  and the wave  $B$  ahead by a phase angle of  $f$

152. Amplitude of a wave is represented by

$$A = \frac{c}{a+b+c}$$

Then resonance will occur when

- (a)  $b = -c/2$
- (b)  $b = 0$  and  $a = -c$
- (c)  $b = -a/2$
- (d) None of these

